Improved Analysis of Vertical Movements in the Carmel Fault Region, Israel, by Extended Free Net Adjustment

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Summary

The Carmel fault is one of the major geological structures in northern Israel. The fault is characterized by potentially hazardous seismic activity. It runs across densely populated areas and in proximity to the Haifa Bay petrochemical factories. Geological and geophysical studies have implied both horizontal and vertical movements along the Carmel Fault. Geodetic study – based on precise leveling that has been measured three times in 1987, 1992 and 2003, using the standard free net adjustment solution – presented significant vertical deformation.

Precise leveling measurements, as other geodetic measurements, partially define the network's datum. The standard free net adjustment solution treats these elements as global geometric parameters of the network, which are defined with a stable datum over time. However, this assumption is not necessarily consistent with reality, and datum elements usually change over time. This makes it necessary to improve the solution, using an extended free net adjustment constraints analysis. The current paper presents an improved analysis of the same precise leveling measurements, which enables the cleaning of the leveling measurements from their datum content. This cleaning minimizes the effect of datum elements on the revaluated deformation and produces results that more clearly reflect the geophysical reality.

Zusammenfassung

Die Carmel Bruchlinie ist eine der wesentlichen geologischen Strukturen im Norden von Israel mit potentiell kritischen seismischen Aktivitäten. Die Bruchlinie verläuft durch dicht besiedelte Gebiete und in der Nähe der Haifa Bay petrochemischen Fabriken. Geologische und geophysikalische Studien postulieren sowohl horizontale als auch vertikale Bewegungen entlang der Bruchlinie. Auf Präzisionsnivellement basierende geodätische Untersuchungen (drei Messkampagnen: 1987, 1992 und 2003) zeigen nach einer freien Standard-Netzausgleichung signifikante vertikale Deformationen.

Präzisionsnivellement-Messungen definieren nur zum Teil das Datum des Netzes. Die freie Standardnetzausgleichung behandelt diese Elemente als globale geometrische Parameter welche mit einem über die Zeit stabilen Datum definiert sind. In der Realität ist diese Annahme nicht notwendigerweise zutreffend und Datumselemente verändern sich über die Zeit. Hieraus ergibt sich die Notwendigkeit die Ausgleichungslösung zu verbessern, indem ein erweiterter Ausgleichungsansatz (mit Zusatzbedingungen) entwickelt wird. Der Artikel präsentiert einen verbesserten Ansatz zur Analyse der vorliegenden Nivellement-Messungen, welcher die Befreiung der Messungen von ihrem Datumsanteil ermöglicht. Diese Befreiung minimiert den Einfluss von Datumselementen auf die zu bewertende Deformation und produziert Ergebnisse, die der geophysikalischen Realität eher entsprechen.

Key words: deformation analysis, geodetic control network, extended free network solution, Carmel Fault, Northern Israel

1 Introduction

The Dead Sea Rift, which separates the Arabian plate from the African plate, dominates the geology of Israel. It extends from the Red Sea in the south through the Dead Sea, and north along the Jordan Valley, the Sea of Galilee and north through the Baqqa Valley in Lebanon towards the Taurus Mountains in Turkey. Its influence on the

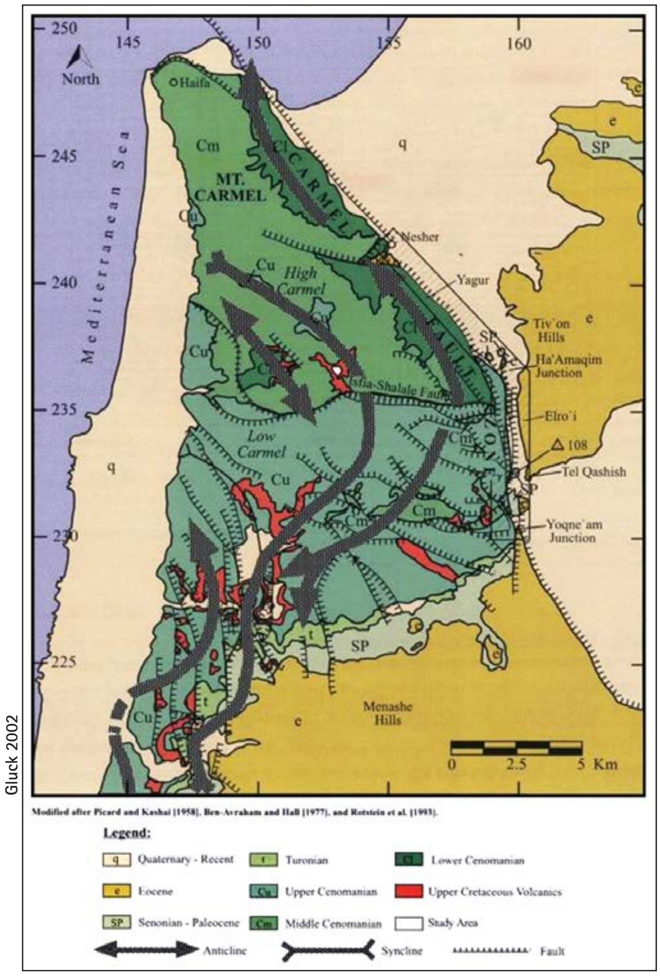

Fig. 1: Geological map of the Carmel Mountain and its surroundings

geological structure in northern Israel is strongly evident in the secondary faulting running in a general northwesterly direction (Schattner 2006). One of the faults is the Carmel fault which separates the Carmel Mountain from the western Lower Galilee (Fig. 1). It is characterized by intense, continuous and potentially hazardous seismic activity (Hofstetter et al. 1996). The fault runs across the city of Haifa and passes close to the petrochemical factories of the industrial area of Haifa Bay. The fault area is known to be of complex geological and tectonic structure (Ron et al. 1991).

The Carmel Fault is a left lateral strike-slip fault, with offset of about 3 to 4 km over the last 20 million years. It is divided into three distinct segments (Rotstein et al. 1993), which can be seen in Fig. 1. The southern segment strikes NW-SE and is characterized by a simple strike-slip motion, with deformation limited to a distinct fault zone. East of the Carmel Mountain, the fault assumes an N-S trend with intense deformation in a zone several kilometres wide. Further to the northwest, the fault resumes its NW-SE trend with decreased deformation. In the middle segment, a component of E-W compression exists as a result of the change in fault strike, and it is associated with active uplifting (Rotstein et al. 1993). The amount of vertical movement is about 1000 m. A major part of this movement occurred during the last two million years; therefore, it was suggested that the fault movement has accelerated. Recent research shows equivocal results concerning the direction of the fault movement (Shahar 2012).

The last serious earthquake in the area occurred in 1984 and was defined with a magnitude of 5.3. The epicenter of this earthquake was located 10 km east of the Carmel fault. Other earthquakes, defined by local epicenters with lower rates of magnitude, are frequently sensed in this region. According to the seismic records, it appears that most seismic activity occurs deep underground, opposite the fault's middle segment.

Geodetic monitoring networks are used to extract geodynamical quantities, such as displacements, velocities, and strains. The free network adjustment techniques developed by Meissl (1969) play a major role in the analysis of deformation networks. Geodetic measurements can, in general, define the inner geometry of the points in the network, but they are incapable of completely determining its datum. For example, the defect of a three-dimensional geodetic network is three when the definition of origin is missing, and it can grow to seven when the definition of the rotations and scale are missing as well. Geodetic measurements contain part of the datum definition. For example, leveling measurements in a network can define its datum parameter of scale, whereas the height differences contribute to defining the relative vertical positions of points in the network. The remaining datum parameters identified with the datum defect of the observational system are defined by imposing an equal number of linear constraints on the estimated coordinate corrections (Koch 1999). Conflicts and apparent discrepancies between the datum content of different observational systems can be analyzed and effectively controlled by the application of the extended free network adjustment approach. Calibration of the measurement instruments right before or following the measurement campaign can reduce the influence of the datum content in the measurements on the deformation parameter. But, with the inevitable improvement in measurement accuracy, the need to use datumless measurements becomes essential (Even-Tzur 2011). Free net adjustment has been employed extensively in time dependent analysis of geodetic networks and has been used as a means for solving the inherent datum problem of the geodetic network (Papo 1985).

Precise leveling is a highly accurate method of geodetic measuring. The height differences between control points can be determined with a standard deviation of less than 1 mm per 1 km; therefore, this method is used for monitoring vertical movements in deformed areas. The limitation of this method is its slow measuring process; only 1 to 2 km of leveling line can be measured per day, making it relatively expensive. A comparison of precise leveling data measured at different times has been made in diverse areas of the world, in order to quantify recent vertical tectonic movements (e.g. Giménez et al. 1996, Kostoglodov et al. 2001, Mäkinen et al. 2003, Yalcinkaya 2003, Ozener et al. 2013).

2 Standard free net adjustment analysis based on precise leveling

2.1 The leveling network in the Carmel fault area

A leveling line containing 31 control points (benchmark, BM) was established in the Carmel Mountain. The line was measured twice by the Survey of Israel (SOI) in 1987 and 1992. The geo-mechanical stability of the BMs is an important condition for obtaining reliable leveling results (Karcz et al. 1976), and so BMs were pinned only on exposed solid bedrock. The leveling line starts near the coastline on the west side of the Carmel Mountain near Atlit (point 719A) and across the mountain up to the city of Nesher (point 4905), located on the east side of the Carmel Mountain (Fig. 2). The leveling line was measured according to the rules of the International Association for Geodesy (Bomford 1980). The leveling line across the Carmel Mountain was measured once again in 2003 by the SOI, in order to continue monitoring vertical movements in the region. In order to make the monitoring more efficient and enhance the detection resolution for vertical deformation, the leveling line was extended across the Carmel fault to the western Lower Galilee (Fig. 1). The extended part of the leveling line was composed of BMs from the national vertical network of Israel.

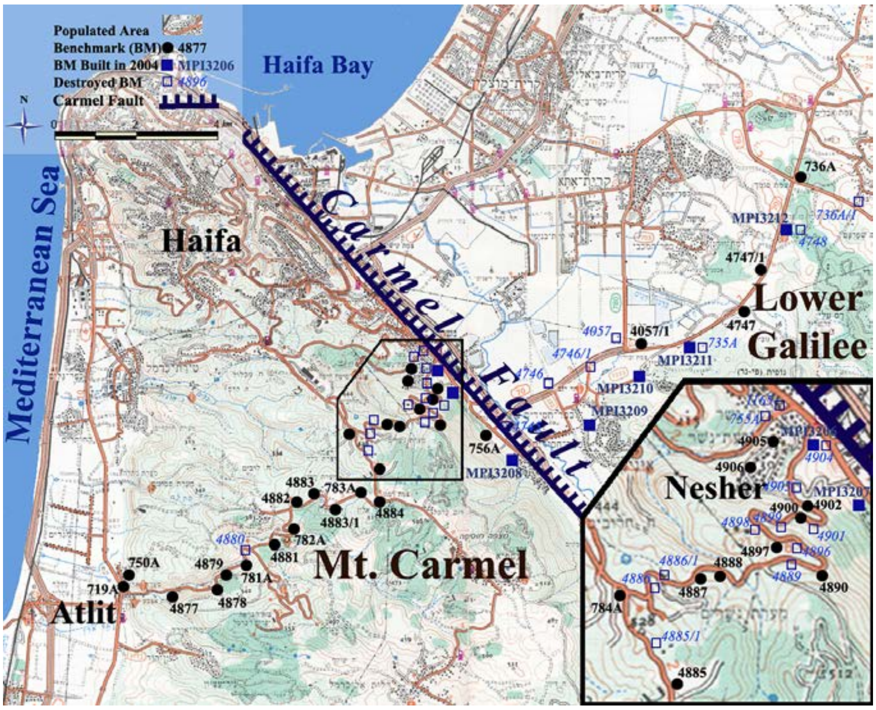

Fig. 2:
Map of the precise leveling network. The map represents the locations of benchmarks on the leveling line between Atlit, on the west side and the Lower Galilee on the eastern side.

Precise leveling measured by the SOI in 1987 in the extended part was used, in order to utilize the measuring data from 2003 for movement analysis. That data was taken from the original books kept in the SOI archive.

2.2 Result of the standard free net adjustment analysis

Shahar and Even-Tzur (2006) presented the standard free net adjustment analysis that solves the inherent datum problem of a leveling network. The analysis was performed based on three measurement campaigns that were carried out between the years 1987 to 2003. An adequate leveling measurement procedure reduces systematic errors (Bomford 1980). Lallemand's method helps to eliminate some other systematic errors (Lallemand 1889). The standard free net adjustment analysis indicates a significant rising of the Carmel Mountain at a rate of up to 2 mm per year, in comparison with the western Lower Galilee, and at a rate of about 1 mm per year in comparison with its north-western slopes. This analysis uses a 5 % significance level as a standard. In addition, the analysis indicates changes over time of the tendency of the western Carmel slopes compared to the ridge. The leveling results indicate a relative sinking of the western slopes in comparison with the Carmel ridge between 1987 and 1992, and an uplifting of them during 1992 to 2003. The rate of estimated uplifting in the second period completely cancels the sinking registered during the previous period, and even indicates a moderate uplifting of the upper western slopes at a rate of up to 0.3 mm per year. These presented results had been accepted in relation to two groups of points that serve as two independent datums. These datums' contents include few points that pass the statistical test which indicates their inner stability (Cooper 1987). The Carmel western slopes datum includes only 25 % of the BMs (3 of 12 BMs) that are located in this area, and the Carmel ridge datum includes only 50 % of the BMs (2 of 4 BMs) that are located on the ridge.

3 Methodology of extended free net adjustment analysis

3.1 Characteristics and disadvantages of the standard free net adjustment analysis

Systematic errors in geodetic measurements may bias the data and affect the parameter values derived from them. Geodetic measurements provide only partial definitions of a geodetic network's datum. Datum components that are not defined by measurements constitute the datum defect (Papo and Perelmuter 1981). A datum defect expresses the partial absence of a reference system and enables an infinite number of trivial constrained solutions. A reference system consists of three components: origin, orientation and scale factor. Geodetic measurements define only part of the datum components of a geodetic network. In monitoring geodetic networks the datum shall be defined with regard to position and movement parameters defined by the mathematical model. A first-order model (kinematic) describes the position and velocity of network points. Therefore, a double reference system is needed: one for positioning and one for velocities. A second-order model (quadratic) then defines, in addition, the defect for acceleration, and so on. Since the position axis of the network points is perpendicular to the time axis, there is no interdependency between the datum definition of the coordinates of the network points and the velocity of the points (Even-Tzur 2010). Consequently, changes in the datum definition of position do not affect the velocity. Therefore, the constraints that apply to the network points can be different from those applied to the velocities (Even-Tzur 2010). In the case of a precise leveling network, the measurements define the scale factor and the gravity defines the orientation.

In this case of a one-dimensional network, the datum defect is characterized by an absence of origin only. A first order model doubles the defect and defines the absence of both the standard network origin and the velocities field origin. Therefore, the datum defect is equal to two ($d = 2$). A second order model also defines the absence of the accelerations field. Therefore the datum defect is equal to three ($d = 3$).

The datum defect can be dealt with by imposing minimum constraints that complete the normal matrix rank (Perelmuter 1979). Although this imposition of arbitrary constraints solves the problem of the datum defect, it results in a bias that is not dependent upon the measurements. A trivial solution, based on a minimal definition of arbitrary constraints, does not necessarily correctly describe the network dynamics and constitutes an example of this problem (Wolf 1977). Free networks are not affected by errors that are not caused by the measurements (Papo 1986). A free network defines each of its points as a datum and expresses the internal geometry and internal precision in relation to its center of gravity. When monitoring crustal deformation, the geodynamic reality usually requires the definition of a local datum and the analysis of the network according to this definition (Papo 1999).

Precise leveling measurements, as other geodetic measurements, partially define the network's datum. In the case of first order vertical network, measured by means of precise leveling, the defect of the datum is expressed by the origin of the position and velocity fields ($d = 2$). To achieve a trivial solution, we imposed two constraints that completed the normal matrix rank, using a point located on the west edge of the leveling line (Fig. 2).

3.2 Extraction of the geometric parameter using the extended free net adjustment solution

An extended free net adjustment solution includes the estimation of the global geometric characteristics of the network points, in addition to their coordinates. As noted above, raw measurements define datum elements, which are deterministic geometric properties of the network at a given time. The extension of the unknown network parameters deals with the inherent datum problem of geodetic networks. The extension enables the simultaneous estimation of both the parameter values that describe the network's global geometric behavior and the coordinates (Papo 1986). The additional parameters enable a distinction between the network's global geometric behavior and the network coordinates of each measurement campaign. This distinction minimizes the effects of global behavior on the solution. The method is based on the mapping of the estimated coordinates of network points. The mapping is performed from a vector space in which the coordinates are affected by datum components, which themselves are defined by measurements, into a vector space in which they are free of any datum effect. The observation equations describe the relationship between the vector l of n measurements, the vector v of residuals, the vector x of u unknowns and the design matrix A as defined by the Gauss-Markov linear model (Koch 1999):

$$Ax = v + l. \tag{1}$$

Let us define d as the size of the datum defect. Therefore, a trivial solution, using minimum constraints, divides the design matrix and the solution vector into two parts:

$$A = \begin{bmatrix} A_{0[n,d]} & A_{1[n,u-d]} \end{bmatrix}$$
$$x = \begin{bmatrix} x_{0[d,1]} \\ x_{1[u-d,1]} \end{bmatrix}. \tag{2}$$

A_0 is an n by d matrix and represents the coefficients of the constrained parameters and x_0 represents their estimate. A_1 is an n by $(u-d)$ matrix that represents the coefficients of other parameters and x_1 represents their estimate.

An extended system operates in a different space. Therefore, it is necessary to use a bijective mapping function for mapping the solution from the origin space X to the target space Z

$$f(y): \mathrm{X} \rightarrow \mathrm{Z} \tag{3}$$

where y represents the global parameters. The extension of the solution defines an extended solution vector z in the target space

$$z = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_0 \\ z_1 \\ y \end{bmatrix} = \begin{bmatrix} w \\ y \end{bmatrix} \tag{4}$$

where the sub-vector z_0 represents the constrained unknowns and z_1 represents the other estimated parameters. Both of these sub-vectors are represented by w and actually represent the adjusted coordinate of each measurement campaign. The sub-vector $z_0 = y$ describes the parameters that define the network's global deformation. This sub-vector may include a variation component of a geometric element. That is to solve y in relation to a stable datum in the way detailed in Even-Tzur (2011).

Therefore, the solution vector can be defined as a Taylor series

$$z = f\left(w^0, y^0\right) + \frac{\partial x}{\partial w} w + \frac{\partial x}{\partial y} y = z^0 + Dw + Fy \tag{5}$$

where z_0 is the approximate solution. D and F are Jacobian matrices that are computed around z^0. Matrix D defines the mapping function and is called the deformation matrix.

The mapping process defines the extended design matrix C:

$$C = A[D \quad F]. \tag{6}$$

Hence, the extended form of the Gauss-Markov linear model is

$$l + v = A\left[Dw + Fy\right] = A[D \quad F]\begin{bmatrix} w \\ y \end{bmatrix} = Cz. \tag{7}$$

The definition of the mapping function as a bijective matrix defines the Jacobian matrices as full rank when

$$\operatorname{rank}(D) = u, \quad \operatorname{rank}(F) = g \tag{8}$$

where g denotes the number of parameters included in y. The dimensions of the extended system must fulfill the conditions

$$g < u - d < n. \tag{9}$$

The datum defect d defines the rank of the extended design matrix

$$\operatorname{rank}(C) = u - d. \tag{10}$$

The null space dimension of the extended design matrix defines the number of linear constraints for a minimum constraints solution

$$\operatorname{nullity}(C) = g + d. \tag{11}$$

A unique minimum constraints solution (Meissl 1969), which satisfies the condition $w^T w \rightarrow \min$, is obtained if

$$H^T w_{\text{unique}} = 0_{[u+g,1]} \tag{12}$$

where H is the geometrical weighted constraints matrix of Helmert (Cooper 1987) that defines the null space of matrix C (Shahar and Even-Tzur 2012):

$$H = \left[H_{0[u,d]} \quad H_{1[u,g]}\right]. \tag{13}$$

The sub-matrix $H_{02[g,d]}$ (as the bottom part of the right sub-matrix $H_{0[u,d]}$) and the left sub-matrix $H_{1[u,g]}$ refer to the extension parameters. Thus, the matrix H is in a special form, where $g = 0$.

The null space (kernel) of matrix C is defined as

$$\operatorname{Ker}(C) = \left\{ z \in R^{u+g} : Cz = \left[AD \quad AF\right]\left[x \quad y\right]^T = 0 \right\}$$

$$\operatorname{Ker}(C) = \left[AD \quad AF\right]\begin{bmatrix} H_{[u,d+g]} \\ 0_{[g,d]}; I_{[g,g]} \end{bmatrix}$$

$$= \left[AD \quad AF\right]\begin{bmatrix} D^{-1}H_{0[u,d]} & D^{-1}H_{1[u,g]} \\ 0_{[g,d]} & I_{[g,g]} \end{bmatrix}. \tag{14}$$

3.3 Extended free net adjustment analysis of a control network in two steps

The simultaneous evaluation of the additional y parameters, along with the points' coordinates, effectively minimizes the effects of the global parameters on the raw measurements. In fact, the extended free net adjustment analysis method can be used to calculate the geometric parameters, simultaneously or in steps. The Two-Step Analysis (Papo and Perelmuter 1993) is a useful and convenient method of solution, which is performed in two stages. When using this method, the first step includes the adjustment of measurement campaigns to static independent networks. Therefore, this step does not include any reference to network changes that have occurred during monitoring. During the second step, the results obtained in the first step serve as pseudo-observations, which are used to adjust an approximate movement model. In fact, the network's deformations are examined at this step, according to this model. This method can also be used for an extended free net adjustment solution. Identical results will be obtained using all methods; however, analysis performed in steps has several advantages over simultaneous analysis. The advantages of a solution performed in steps are well expressed, even in an extended free network adjustment solution (Papo and Perelmuter 1993).

3.4 Cleaning of leveling measurements

The quality of a control network and its ability to monitor deformations depends on the quality of the raw measurements, so it is essential to be able to process reliable raw measurements. This processing includes the detection of measurements that contain gross errors and the implementation of compensation procedures to resolve systematic errors. Geodetic measurements define, as noted, a partial definition of the network's datum. We treat these elements as global geometric parameters of the network, which are defined with stable datum over time. However, this assumption is not necessarily consistent with physical reality, and datum elements, which are defined by the measurements, may sometimes change over time. This situation may lead to the smearing of the systematic error's components into the parameter estimation system and may even be interpreted as a deformation (Wolf 1977). Using an extended free net adjustment constraints solution enables differentiation of the measurements and the datum elements. This differentiation eliminates the effect of these elements on the revaluation of the network solution.

The deterministic ingredient of a 1D leveling network includes the scale factor element (SF), which is defined by measurement, therefore, we define $g = 1$. The datum defect consists of only the absence of origin definition. This defect is defined for location, so $d = 1$. Mapping of the measurement to the vector space of the extended free net adjustment solution is performed by the mapping function (Shahar and Even-Tzur 2012)

$$f(y): \mathrm{X} \rightarrow \mathrm{Z}$$
$$f(y): x \rightarrow w \cdot SF \quad (15)$$

where the extended solution vector z is defined as:

$$z = \begin{bmatrix} z_0 & z_1 & y \end{bmatrix}^T = \begin{bmatrix} w & y \end{bmatrix}^T$$
$$y = SF$$
$$x_0 = z_0 \cdot y = z_0 \cdot SF$$
$$x_1 = z_1 \cdot y = z_1 \cdot SF. \quad (16)$$

The deformation matrix is defined as

$$D = \frac{\partial f(y)}{\partial w} = \frac{\partial x}{\partial w} = SF \cdot I. \quad (17)$$

I is an Identity matrix $[u,u]$. The coefficient matrix F is actually a vector of u parameters:

$$F = \frac{\partial f(y)}{\partial y}. \quad (18)$$

The null space is defined in this case by the sub matrices

$$H_{00} = 1 \quad ; \quad H_{10} = -h_1$$
$$H_{01} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad ; \quad H_{11} = \begin{bmatrix} -h_2 \\ -h_3 \\ \vdots \\ -h_{u-1} \end{bmatrix}$$
$$H_{02} = 1 \quad ; \quad H_{12} = -h_u \quad (19)$$

where h_i is the height of point i.

The extended revaluation cleans the measurements by extraction of the systematic errors caused by the datum elements, which are defined by the measurements. This extraction characterizes the measurements without the bias of datum elements. However, the revaluated solution is a free network and as such, all network points are defined as a datum. Since the solution is obtained for each measurement campaign, it is necessary to determine a fixed set of points, defined with the same approximated values, for all the measurement campaigns. Revaluation in this way ensures the compatibility of the cleaned parameters, which refer to the same reference source, and enables an optimal cleaning of the datum element. However, cleaning using a free network assumes the absence of deformation along the measurement campaigns. This assumption is mistaken in the case of a deformable network and the solution shall be revaluated according to a stable datum, using a weight constraints solution procedure. The revaluation of geometric parameters is performed during the first step of the network solution. In this step, the revaluated networks are static for each campaign, with no definition of the time dimension. Therefore, the datum defect is solved only for the position.

3.5 Analysis of vertical deformation

The standard free net adjustment solution treats the datum elements as global geometric parameters of the network, which are defined with a stable datum over time. However, this assumption is not necessarily consistent with reality, and datum elements may change over time. This is why it is necessary to improve the solution, using an extended free network adjustment constraints analysis. In this stage of monitoring, where the network is measured in three campaigns only, we use a first order kinematic model. The height of a point ς_i in time t is described in the linear model as

$$\varsigma_i = \varsigma_0 + \dot{\varsigma}_i (t - t_0) \quad (20)$$

where ς_0 is the height of the point at standard time t_0 (the reference height) and $\dot{\varsigma}_i$ is the vertical velocity of the point.

The datum defines the reference frame of the geodetic network and is a part of the network, which remains stable during the monitoring campaigns. Therefore, deformation and movements are defined relative to the datum points. We use statistical tools to define the particular part of the network that can serve as a datum at a given significance level. A datum must be located in an area that has been defined as internally stable, according to available geological knowledge and sensibleness. The improved analysis is based on two stable areas that serve as datums:

1. The Carmel ridge,
2. the Carmel western slopes.

A transformation between one solution pertaining to a specific datum into another solution pertaining to another datum can be performed by S-transformation (Baarda 1973) using a transformation matrix $J = f(H)$ where for the leveling network $H = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$. An extended S-transformation is needed for an extended solution, using an extended transformation matrix J_{ex}, which is defined as follows (Even-Tzur 2011):

$$J_{\mathrm{ex}} = f(H, D, F). \quad (21)$$

The free network solution and its covariance matrix are obtained using

$$w_{\mathrm{free}} = J_{\mathrm{ex}} w_{\mathrm{unique}}$$
$$\Sigma_{w(\mathrm{free})} = J_{\mathrm{ex}} \Sigma_{w(\mathrm{unique})} J_{\mathrm{ex}}^{\ T}. \quad (22)$$

Basically, a matrix of weight constraints P_x including diagonal constraints components defined by the digit one, while the components of the other matrix are defined by zeros, should be used (Wolf 1977). This matrix defines the S-transformation matrix $J_{\text{ex(weighted)}}$ while

$$J_{\text{ex(weighted)}} = f\left(H, D, F, P_x\right) \quad (23)$$

enables the calculation of an extended weighted datum free net solution and its covariance matrix

$$w_{\text{weighted}} = J_{\text{ex(weighted)}} w_{\text{free}}$$
$$\Sigma_{w(\text{weighted})} = J_{\text{ex(weighted)}} \Sigma_{w(\text{free})} J_{\text{ex(weighted)}}^T. \quad (24)$$

3.6 Statistical tests

Hypothesis tests are normally used to estimate the fitting of the motion model and its significance. The selection of the datum points involves a few considerations. First of all, the chosen group of points has to be on stable and homogeneous ground. Second, the group of points must satisfy the null hypothesis

$$\text{H}_0 : \dot{\varsigma}_1 = \dot{\varsigma}_2 = ... = \dot{\varsigma}_r = 0. \quad (25)$$

While aside this

$$\text{H}_1 : \dot{\varsigma}_1 \neq 0 \,\|\, \dot{\varsigma}_2 \neq 0 \,\|\, ... \,\|\, \dot{\varsigma}_r \neq 0 \quad (26)$$

where r is the number of leveling lines between the benchmarks that have been chosen to serve as a datum.

We will reject H_0 with a significance level of α if

$$\frac{|\dot{\varsigma}_1|}{\sigma_1} > Z_{1-\frac{\alpha}{2}} \,\|\, \frac{|\dot{\varsigma}_2|}{\sigma_2} > Z_{1-\frac{\alpha}{2}} \,\|........\|\, \frac{|\dot{\varsigma}_k|}{\sigma_k} > Z_{1-\frac{\alpha}{2}}. \quad (27)$$

The meaning of accepting the null hypothesis H_0 is that the datum points are stable to each other at a significance level of α. The significant velocity of a point, in relation to an acceptable datum, has to be tested in a similar way for a single point $r = 1$.

4 Results

As aforesaid, in geodetic monitoring networks the values of the deformation parameters are estimable only if the datum of the network has not been changed between measurement epochs. We use the Two-Step method. In the first step, each leveling campaign was adjusted to a network with the same approximated values for the heights of the points. During the second step a first order kinematic model was adjusted. Revaluation in this way ensures the compatibility of the cleaned parameters, which refer to the same reference source, and enables an optimal cleaning of the datum element. We chose the standard free net adjusted heights of the first campaign (1987) to serve as approximate values for the adjustment of all campaigns. These values define the reference SF of the 1987 reference campaign as a fixed value (we set this value as 1). Tab. 1 represents the revaluated SF corrections of the three measurement campaigns in relation to the reference SF. These values were calculated in relation to a stable datum, based on the twelve points located on the Carmel western slopes (see Fig. 2 and 3). These points pass the partial congruence test, which indicates their inner stability (Cooper 1987). This revaluation, in relation to a stable datum, ensures the cleaning of deterministic changes in the SF only, without being affected by any deformations that may have occurred between the campaigns. The revaluated SF value of the second campaign (1992) changed in 44 ppm and the SF value of the third campaign (2003) changed in 26 ppm, related to the reference campaign.

Tab. 1: The estimated scale factor values of the extended free net adjustment solution and their precision. The values are represented after the cleaning of the global deformation was performed in relation to the reference campaign (1987).

Campaign	SF Correction [ppm]	σ [ppm]
1987	0	5.2
1992	44	4.9
2003	26	5.3

The extended free net adjustment solution is presented in Tab. 2, together with the standard free net adjustment solution, with regard to the Western slopes datum. Fig. 3 demonstrates the extended solution. The analysis of the extended free net adjustment shows the stability of all twelve points that are located on the western slopes, without significant movement, while in the standard free net solution, the datum points were defined with inner instability. The analysis shows the sinking of the Carmel ridge at a rate of up to 0.7 mm per year with regard to this datum. In addition, the western Lower Galilee is sinking at a rate of up to 1.6 mm per year with regard to this datum.

The extended free net adjustment solution and the standard free net solution, with regard to the Carmel ridge datum, is presented in Tab. 3. Fig. 4 demonstrates the extended solution. The analysis of the extended free net adjustment shows the stability of all four points that are located on the ridge, without significant movement, while in the free net solution, the datum points were defined with inner instability. The analysis shows that the Carmel western slopes have risen at a rate of up to 1 mm per year with regard to this datum. In addition, the western Lower Galilee is sinking at a rate of up to 1.1 mm per year with regard to this datum.

In addition, using an extended free net adjustment solution makes it possible to base a solution regarding the Carmel north-eastern slopes on a stable datum that includes only 3 points. This datum would be unacceptable when using a free net solution.

5 Summary

Precise leveling, as other geodetic measurements, partially defines the network's datum. Despite the use of existing algorithms for the elimination of systematic errors, the spreading of systematic components into the parameter values estimation system still occurs and is often misinterpreted as indicating the inner instability of homogeneous areas, which otherwise would be considered suitable to serve as stable datums. We ascribe these perceived instabilities to the effects of the geometrical deterministic component, SF in this case of measurements, which spreads deterministic behavior to the standard free net adjustment solution.

This study demonstrates the successful use of the extended free net adjustment solution algorithm in the case of a vertical control network, based on precise leveling measurements, in the Carmel fault region. The leveling line, containing 31 BMs, starts at the coastal plain on the western side of the Carmel Mountain and crosses the Carmel ridge and the Carmel active fault and continues up to the western mountains of the Lower Galilee. The line has been measured three times, in 1987, 1992 and 2003. The extended free net adjustment solution has been shown to enable reliable analysis based on acceptably stable datums, when compared to the use of a free net solution.

Using this algorithm, the analysis cleans SF differences of up to 44 ppm between measurements taken using different instruments during the different campaigns. This cleaning minimizes systematic error caused by these differences – error which has not previously been taken into account using the standard tools. This new algorithm optimizes results and enhances the ability of the network to detect realistic

Tab. 2: The velocity field of the free leveling network weighted with the Carmel western slopes datum and its precision. The result is represented as velocities according to the linear model in the different time sections and compared with its precision (mm per year) at significance levels of 5 % and 1 %. A significant velocity at a level of α % signed with a deep background of its precision at this level.

Solution		Extended free net			Free net		
Point	Datum	Velocity	0.05	0.01	Velocity	0.05	0.01
719A	Datum	0.03	0.2	0.3	−0.18	0.2	0.3
4877	Datum	−0.09	0.2	0.2	−0.26	0.2	0.2
4878	Datum	0.00	0.1	0.2	−0.09	0.1	0.2
4879	Datum	0.00	0.1	0.2	−0.08	0.1	0.2
781A	Datum	0.00	0.1	0.2	−0.05	0.1	0.2
4881	Datum	−0.02	0.1	0.1	−0.06	0.1	0.1
782A	Datum	0.07	0.1	0.1	0.09	0.1	0.1
4882	Datum	0.07	0.1	0.2	0.13	0.1	0.2
4883	Datum	0.12	0.1	0.2	0.19	0.1	0.2
48831	Datum	0.01	0.1	0.2	0.12	0.1	0.2
783A	Datum	−0.02	0.2	0.2	0.16	0.16	0.2
4884	Datum	−0.16	0.2	0.2	0.02	0.2	0.2
4885		−0.29	0.2	0.2	−0.10	0.2	0.2
784A		−0.34	0.2	0.3	−0.20	0.2	0.3
4887		−0.55	0.3	0.3	−0.52	0.3	0.3
4888		−0.65	0.3	0.4	−0.64	0.3	0.4
4890		−0.94	0.3	0.4	−0.96	0.3	0.4
4897		−0.65	0.3	0.4	−0.73	0.3	0.4
4900		−0.66	0.3	0.4	−0.82	0.3	0.4
4902		−0.71	0.3	0.5	−0.92	0.3	0.5
4906		−0.80	0.4	0.5	−1.00	0.4	0.5
4905		−0.65	0.4	0.5	−0.91	0.4	0.5
756A		−1.01	0.4	0.5	−1.35	0.4	0.5
4747		−0.33	0.5	0.6	−0.66	0.5	0.6
47471		−0.85	0.5	0.6	−1.13	0.5	0.6
736A		−1.61	0.5	0.6	−1.85	0.5	0.6

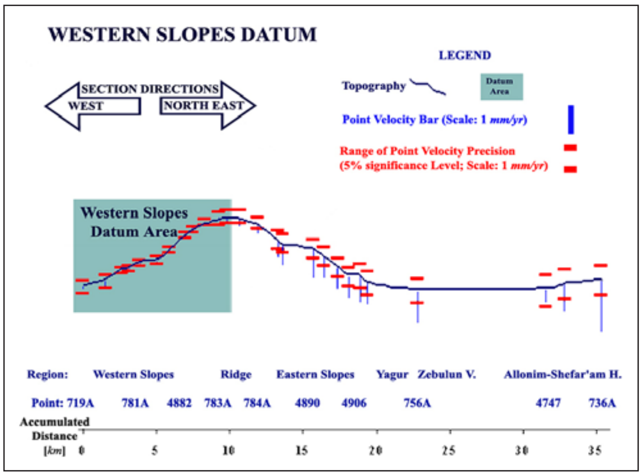

Fig. 3: Extended solution of the leveling network weighted by the Carmel western slopes datum. A significant BM velocity (mm per year) appears where the velocity line deviates from the precision margins at a 5 % significance level.

Tab. 3: The velocity field of the free leveling network is weighted with the Carmel ridge datum and its precision. The result is represented as velocities, according to the linear model in the different time sections and compared with its precision (mm per year) at significance levels of 5 % and 1 %. A significant velocity at a level of α % signed with a deep background of its precision in this level.

Solution		Extended free net			Free net		
Point	Datum	Velocity	0.05	0.01	Velocity	0.05	0.01
719A		0.99	0.4	0.5	0.19	0.4	0.5
4877		0.82	0.3	0.4	0.11	0.3	0.4
4878		0.83	0.3	0.4	0.27	0.3	0.4
4879		0.80	0.3	0.4	0.28	0.3	0.4
781A		0.76	0.3	0.4	0.31	0.3	0.4
4881		0.71	0.3	0.4	0.31	0.3	0.4
782A		0.72	0.3	0.3	0.46	0.3	0.3
4882		0.58	0.2	0.3	0.49	0.2	0.3
4883		0.57	0.2	0.3	0.56	0.2	0.3
48831		0.41	0.2	0.2	0.49	0.2	0.2
783A		0.34	0.2	0.2	0.53	0.2	0.2
4884		0.18	0.1	0.2	0.39	0.1	0.2
4885	Datum	0.06	0.1	0.2	0.27	0.1	0.2
784A	Datum	0.05	0.1	0.1	0.17	0.1	0.1
4887	Datum	–0.02	0.1	0.1	–0.16	0.1	0.1
4888	Datum	–0.09	0.1	0.1	–0.28	0.1	0.1
4890		–0.34	0.1	0.2	–0.60	0.1	0.2
4897		0.02	0.2	0.2	–0.36	0.2	0.2
4900		0.10	0.2	0.3	–0.45	0.2	0.3
4902		0.11	0.2	0.3	–0.55	0.2	0.3
4906		0.02	0.3	0.3	–0.64	0.3	0.3
4905		0.23	0.3	0.4	–0.54	0.3	0.4
756A		–0.09	0.3	0.4	–0.99	0.3	0.4
4747		0.59	0.4	0.5	–0.29	0.4	0.5
47471		0.03	0.4	0.5	–0.76	0.4	0.5
736A		–0.74	0.4	0.5	–1.48	0.4	0.5

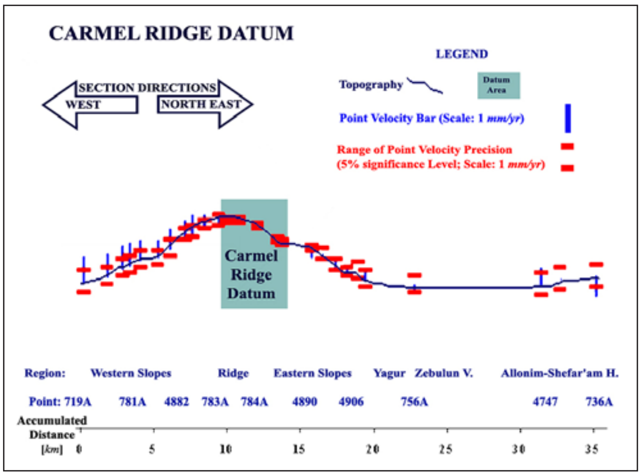

Fig. 4: Extended solution of the leveling network weighted by the Carmel ridge datum. A significant BM velocity appears where the velocity line deviates from the precision margins at a 5 % significance level.

and reliable deformations. The analysis shows a significant sinking of the Carmel ridge at a rate of up to 1 mm per year relative to the western slopes of the Carmel, and a rising of the ridge at a rate of up to 0.7 mm per year relative to the Lower Western Galilee.

Using the extended free net adjustment solution, all 12 points located on the western slopes of the Carmel, as well as all the points located on the Carmel ridge, have been revaluated to indicate inner stability and to behave as two homogeneous units. This solution enables an improved analysis, based on a wide stable datum, and yields results that more accurately reflect the geophysical reality.

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