Accuracy Improvement of Stochastic Modeling of Inertial Sensor Errors

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Summary
In the last decade, the utilization of an Inertial Navigation System (INS) as a stand-alone system or integrated with other navigation systems such as a Differential Global Positioning System (DGPS) has become a standard tool in many applications. However, current INS error models that are used in most INS and INS/DGPS applications have some limitations, which in turn affect the overall navigation accuracy. One of these limitations is associated with the stochastic modeling of inertial sensor errors in the INS error model. For most of the navigation-grade INS systems (gyro drift 0.005–0.01 deg/h), a 1st order Gauss-Markov (GM) model is usually used. This is also true for low-cost inertial systems (gyro drift 100–1000 deg/h), although sometimes a white noise process instead of a 1st order GM model is implemented. In this Paper, an overview of the different possible random processes for stochastic modeling of inertial sensor errors is presented. The actual behavior of INS sensor random errors is shown by computing the actual Autocorrelation Sequence (ACS) of inertial experimental data. The results showed that none of the commonly used random processes is adequate for modeling INS sensor errors. In addition, numerical analyses are performed to illustrate the poor accuracy of ACSs that are obtained from inertial experimental data. The paper offers a new method to model the INS stochastic errors using Autoregressive (AR) models of orders higher than one. Using real INS data, the results showed that the performance of AR processes is better than the performance of any of the currently used processes by 40 % to 70 %.

Zusammenfassung
Ein Trägheitsnavigationssystems (INS) als Einzelsystem oder in Verbindung mit anderen Navigationssystemen wie dem Differentiellen Global Positioning System (DGPS) ist im letzten Jahrzehnt ein Standardwerkzeug in vielen Anwendungsbereichen geworden. Jedoch haben Fehlermodelle, die momentan in den meisten INS- oder INS/DGPS-Systemen Verwendung finden, Einschränkungen, die die Genauigkeit der Navigationslösung beeinträchtigen. Eine dieser Einschränkungen ist mit der Modellierung des stochastischen Fehlers im INS-Fehlermodell verknüpft. Für die meisten INS-Systeme der Navigationsklasse (Kreiseldrift 0.005–0.01 deg/h) wird normalerweise ein Gauß-Markov(GM)-Modell erster Ordnung verwendet. Dies gilt auch für billigere Systeme (Kreiseldrift 100–1000 deg/h), wenn auch manchmal ein Prozess mit weißem Rauschen anstelle des Gauß-Markov-Modells erster Ordnung implementiert ist. In dieser Veröffentlichung wird eine Übersicht der verschiedenen möglichen Zufallsprozesse für die stochastische Modellierung eines Trägheitsnavigationssystems vorgestellt. Das tatsächliche, zufällige Fehlerverhalten des INS-Sensors wird mit Hilfe der Autokorrelationssequenz (ACS) von Versuchsdaten bestimmt. Die Ergebnisse zeigen, dass keiner der allgemein verwendeten Zufallsprozesse geeignet ist, das Fehlerverhalten des INS-Sensors zu beschreiben. Um die geringe Genauigkeit von ACSs, die aus Versuchsdaten gewonnen werden, zu verdeutlichen, werden zusätzlich numerische Berechnungen durchgeführt. Darüber hinaus wird hier eine neue Methode zur Modellierung des zufälligen Fehlerverhaltens unter Verwendung von autoregressiven (AR) Modellen höherer Ordnung vorgestellt. Testergebnisse mit gemessenen INS-Daten zeigen, dass die Leistung des autoregressiven Prozesses um 40 % bis 70 % besser als jedes der bisher verwendeten ist.

1 Introduction

The integration of an Inertial Navigation System (INS) with a DGPS has been implemented for several years in different geodetic applications. In all of these applications, the integrated INS/DGPS system is used for providing the navigation information (position and orientation) for the system carrier. For mobile mapping purposes, the INS/DGPS navigation information is provided to an imaging sensor mounted on the same carrier. The imaging sensor can be a frame-based (analog) aerial camera, a Charge Coupled Device (CCD) digital camera, a laser scanner, a pushbroom scanner or a Synthetic Aperture Radar (SAR). Another application of INS/DGPS that has received the attention of geodesists in the last decade is airborne gravimetry. Using the INS/DGPS navigation solution (for the computation and compensation of the system errors) and subtracting the aircraft acceleration (obtained by twice differentiating DGPS positions) from the total sensed acceleration (obtained by INS accelerometer specific force measurements), the gravity field can be determined with high accuracy.

The accelerometer and gyro sensor errors of an INS consist of two parts: a deterministic part and a stochastic part. The deterministic part includes biases and scale factors, which are determined by calibration. The stochastic part is basically due to the inertial sensor residual errors. In the standard operation of INS standalone and INS/DGPS navigation applications, the INS mechanization is described using a system of differential equations that are solved to provide positions, velocities and attitudes. Due to the INS sensor errors, the solution of such equations contains both systematic and stochastic errors.

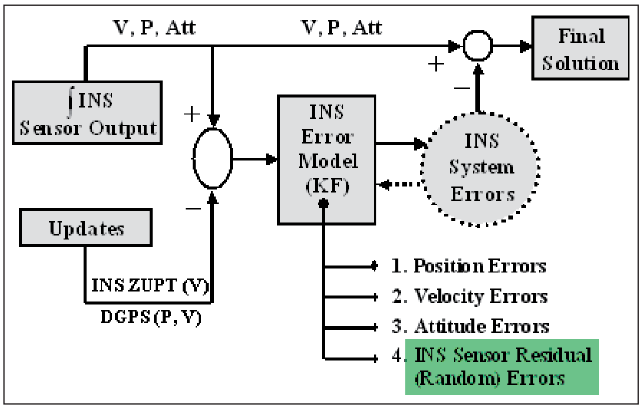

Fig. 1: INS Mechanization and Error Modeling

The INS deterministic error models are obtained by linearizing the INS mechanization equations while the INS stochastic error models are formed for the INS sensor random errors using a random process such as: white noise, random constant, random walk, Gauss-Markov or periodic random processes. Both deterministic and stochastic error models are included in the INS error model so that both error types can be estimated through Kalman Filter (KF), see Fig. 1. However, it is the latter error type that will be revisited in this paper. In Fig. 1, the system initial trajectory (velocity, position and attitude) is obtained by integrating the output of the INS sensors. The updates for the KF are obtained through Zero-Velocity Updates (ZUPTs) in case of INS stand-alone navigation and DGPS position and velocity in case of INS/DGPS integration.

In most of the current INS error models, the inertial sensor random errors (residual biases b) are described by a random process, where the process is considered to be stationary in general (it will be shown later that this assumption is not always valid). In this case, the process is assumed to be completely defined by its Autocorrelation Function (ACF) specifications (Brown and Hwang 1997). For a stationary random process b(t), the ACF is defined as:

$$\bar{\mathbf{R}}_{bb}(\tau) = \mathbf{E}[b(t).b(t+\tau)] = \text{mean}\left[\sum_{t=-\infty}^{\infty} b(t).b(t+\tau)\right], \qquad (1)$$

where E[] is the mathematical expectation operator, t is the time and τ is the time lag between samples. For discrete-time signals, the Autocorrelation Sequence (ACS) is computed instead. The ACS is defined by replacing t by a sampling sequence k and τ by a sampling lag m, and hence:

$$\bar{\mathbf{R}}_{bb}(m) = \mathbf{E}[b(k).b(k+m)] = \text{mean}\left[\sum_{k=-\infty}^{\infty} b(k).b(k+m)\right]. \qquad (2)$$

The values of $\bar{\mathbf{R}}_{bb}(m)$ are known as the ensemble autocorrelations since it is assumed to be for infinite data records. In practice, the ACS is computed for a finite data of length N, and thus, $\bar{\mathbf{R}}_{bb}(m)$ is replaced by the sample autocorrelations $\mathbf{R}_{bb}(m)$ (Orfanidis 1988). Therefore, for a time-series of measurements b(k), k = 1, 2, 3, …, N, the sample ACS is determined by:

$$\mathbf{R}_{bb}(m) = \mathbf{E}[b(k).b(k+m)] = \frac{1}{N-m}\sum_{k=1}^{N-m} b(k).b(k+m)\,. \qquad (3)$$

The value of the ACS at lag m = 0 is given as:

$$\mathbf{R}_{bb}(0) = \mathbf{E}[b^2(k)] = \frac{1}{N}\sum_{k=1}^{N} b^2(k) = \sigma_b^2 + \mu_b^2\,, \qquad (4)$$

where σ_b and μ_b are the standard deviation and mean of the residual bias b, respectively. The ACS of the INS sensor errors is computed using a long sequence of INS sensor measurements of static data after removing its mean (i. e. μ_b will be zero). The Fourier Transform of the ACF is the Power Spectral Density (PSD) $\mathbf{S}_{bb}$ (or the Periodogram in case of the ACS).

2 Possible Random Processes for Modeling INS Sensor Errors

2.1 White Noise (WN)

A WN process usually has a zero mean and when stationary, it has a constant PSD $\mathbf{S}_{bb} = \mathbf{S}_{bb}(0)$ (Anderson and Moore 1979). Thus, the ACF and ACS of a stationary WN process are determined as:

$$\mathbf{R}_{bb}(\tau) = \mathbf{S}_{bb}(0).\boldsymbol{\delta}(\tau)\,, \qquad (5a)$$
$$\mathbf{R}_{bb}(m) = \mathbf{S}_{bb}(0).\boldsymbol{\delta}(m)\,, \qquad (5b)$$

where $\boldsymbol{\delta}()$ is the delta function. Recalling Equ. (4) and considering the definition of $\boldsymbol{\delta}(m)$ into Equ. (5b):

$$\mathbf{R}_{bb}(0) = \mathbf{E}[b^2(k)] = \boldsymbol{\sigma}_b^2 = \mathbf{S}_{bb}(0).\boldsymbol{\delta}(0)$$
$$\Leftrightarrow \mathbf{R}_{bb}(m) = \boldsymbol{\sigma}_b^2.\boldsymbol{\delta}(m)\,. \qquad (6)$$

Therefore, a WN process is called sometimes a pure random process (Bryson Jr. and Ho 1975). The ACF of a WN process is shown in Fig. 2. Finally, and taking into

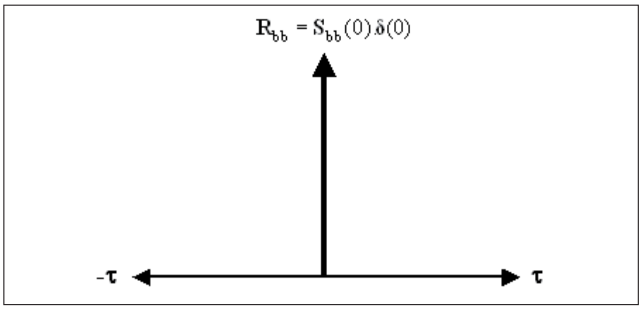

Fig. 2: Autocorrelation Function (ACF) of A White Noise Process

account the definition of $\boldsymbol{\delta}(\tau)$, the variance of a WN process is infinite. This implies that such a process is only a theoretical concept (Andreyev 1969, Newland 1975) or that the process is not physically realizable (Gelb 1974). In spite of that, however, WN can be used successfully to approximate some physical processes. Moreover, and as will be discussed in the following Section, some other random processes are generated by passing a WN sequence through linear filters.

2.2 Shaping Filters

As will be shown later in Section 3, the computed ACS of the INS sensor residual errors does not represent a white sequence process. Instead, these errors can be appropriately modeled by passing a WN w(t) through a certain shaping filter (linear dynamic system) to yield an output of time-correlated noise. This will change the correlation characteristics of the input sequence to fit the actual residual error component. In the following subsections, some random processes that are generated by passing a white sequence through shaping filters are discussed. Special attention will be given to Gauss-Markov processes.

2.2.1 Random Constant (RC)

The RC is an unpredictable random quantity with a constant value (Papoulias 2001). In this case, the sensor residual error b(t) is defined by the following difference equation:

$$b_{k+1} = b_k , \tag{7}$$

where b(k) is written as b_k for simplicity. Substituting Equ. (7) into Equ. (3) results in:

$$\mathbf{R}_{bb}(m) = \mathbf{E}[b_k^2] = \mathbf{R}_{bb}(0) = \text{Const.} \tag{8}$$

Thus, the RC is the special case of a shaping filter with a random initial condition. It is not really a filter, since it is an integration output with no input (Grewal and Andrews 2001).

2.2.2 Random Walk (RW)

For a RW process, the difference $(b_{k+1} - b_k)$ is a white sequence w_k (Shan 2002), i.e.:

$$b_{k+1} = b_k + w_k . \tag{9}$$

Thus, for a very large number of data samples, Equ. (9) converges to:

$$b_{k+1} = \sum_{i=1}^{k} w_i . \tag{10}$$

From Equ. (9), the RW process is generated by integrating uncorrelated random sequences. Using Equ. (10), the mean μ_b of an RW process is equal to zero. Taking into account that w_i are uncorrelated sequences, the variance σ_b^2 is computed as:

$$\sigma_b^2 = \mathbf{E}[b_{k+1}^2] = \mathbf{E}\left[\sum_{i=1}^{k} w_i\right]^2 = \sum_{i=1}^{k} \mathbf{E}[w_i^2] = k\sigma_w^2 . \tag{11}$$

Therefore, the RW process is not stationary since its variance is changing with the number of samples, and hence, the characteristics of the ACS cannot be used to completely define the process (Brown and Hwang 1997). Even though, the difference $(b_{k+1} - b_k)$ itself is stationary.

2.2.3 Gauss-Markov (GM)

GM processes are stationary processes that have exponential ACFs. GM processes are useful in many engineering applications since they can describe many physical random processes with good approximation (Brown and Hwang 1997, Bethel et al. 2000). Most of the present inertial systems model the sensor residual errors as a 1st order GM process with a fairly large correlation time (Schwarz and Wei 2001). The ACF of a zero-mean 1st order GM process is defined as:

$$\mathbf{R}_{bb_1}(\tau) = \sigma_b^2 e^{-\beta_1|\tau|} , \tag{12}$$

where β_1 is the reciprocal of the process correlation time τ_{c_1} ($\tau = \tau_{c_1}$ at $\mathbf{R}_{bb_1}(\tau) = \frac{1}{e}\sigma_b^2$). This ACF is shown in Fig. 3. A 1st order GM process is widely used for modeling INS sensor errors since it has a very simple mathematical description, which makes it easy to implement in the error model. Using a 1st order GM model, the sensor error is defined as (Salychev 2000):

$$b_{k+1} = (1 - \beta_1 \ \Delta t)\ b_k + \sqrt{2\beta_1\sigma_b^2}\ \Delta t\ w_k , \tag{13}$$

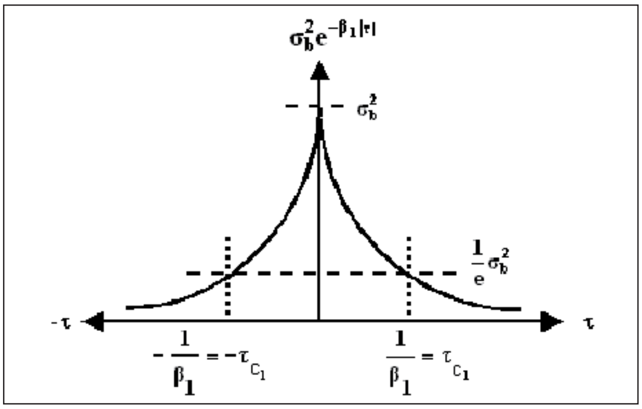

Fig. 3: ACF of a 1st Order GM Process

where Δt is the sampling interval. However, it should be clarified here that the ACF expression of a 1st order GM process [Equ. (12)] was derived by implementing Equ. (1) on Equ. (13) with considering $\tau = \Delta t$ and $\mu_b = \mu_w = 0$.

A family of higher-order GM processes can be generated using Equ. (12). The ACF general formula for a GM process of order p is represented by:

$$\mathbf{R}_{bb_p}(\tau) = \sigma_b^2 e^{-\beta_p|\tau|} \sum_{n=0}^{p-1} \frac{(p-1)!(2\beta_p|\tau|)^{p-n-1}(p+n-1)!}{(2p-2)!n!(p-n-1)!}. \quad (14)$$

However, Equ. (14) is given in Gelb (1974) but the term »$(p+n-1)!$« is missing from the numerator. Thus, the ACF of a GM process of any required order can be obtained from Equ. (14). To compute the correlation time τ_{c_p} in this case, Equ. (14) is solved analytically with the condition: $\mathbf{R}_{bb_p}(\tau_{c_p}) = \frac{1}{e}\sigma_b^2$. The ACF and the corresponding correlation time values for the GM process family are summarized in Tab. 1. To show the graphical characteristics of the ACF of different orders of GM processes, first a constant correlation time is assumed for all orders. Hence, the corresponding β_p is computed for each order p using the formulae in Tab. 1. Then an ACF is generated for each order using Equ. (14). Assuming a data length of 8 hours, Figs. 4a and 4b show the ACF of 1st to 5th order GM processes with a different assumed correlation time for each figure.

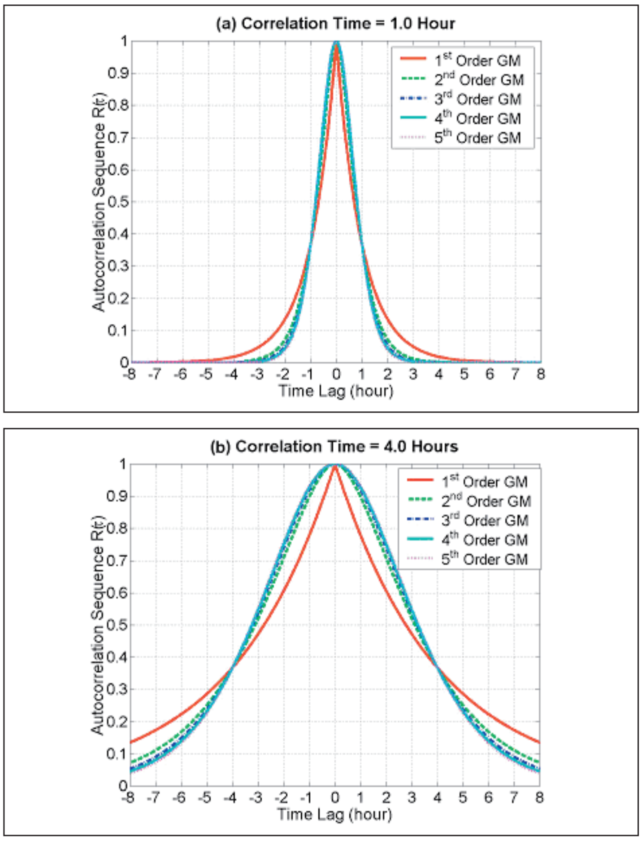

Fig. 4: The Generated ACFs for Different Orders of GM Processes

Tab. 1: The ACFs and Corresponding Correlation Times for Different Order GM Processes

Order p of GM Process	Autocorrelation Function $\mathbf{R}_{bb_p}(\tau)$	Correlation Time τ_{c_p}
1	$\sigma_b^2 e^{-\beta_1|\tau|}$	$\frac{1}{\beta_1}$
2	$\sigma_b^2 e^{-\beta_2|\tau|}\ (1+\beta_2|\tau|)$	$\frac{2.14619}{\beta_2}$
3	$\sigma_b^2 e^{-\beta_3|\tau|}\ (1+\beta_3|\tau|+\frac{1}{3}\beta_3^2|\tau|^2)$	$\frac{2.90463}{\beta_3}$
4	$\sigma_b^2 e^{-\beta_4|\tau|}\ (1+\beta_4|\tau|+\frac{2}{5}\beta_4^2|\tau|^2+\frac{1}{15}\beta_4^3|\tau|^3)$	$\frac{3.51265}{\beta_4}$
5	$\sigma_b^2 e^{-\beta_5|\tau|}\ (1+\beta_5|\tau|+\frac{3}{7}\beta_5^2|\tau|^2+\frac{2}{21}\beta_5^3|\tau|^3+\frac{1}{105}\beta_5^4|\tau|^4)$	$\frac{4.03422}{\beta_5}$
⋮	⋮	⋮
p	$\sigma_b^2 e^{-\beta_p|\tau|} \sum_{n=0}^{p-1} \frac{(p-1)!\ (2\beta_p|\tau|)^{p-n-1}\ (p+n-1)!}{(2p-2)!\ n!\ (p-n-1)!}$	Solved for each p with the condition $\mathbf{R}_{bb_p}(\tau_{c_p}) = \frac{\sigma_b^2}{e}$

2.2.4 Periodic Random (PR)

The ACF for random processes that are known to have periodic behavior is represented by an exponential and periodic functions, such as:

$$\mathbf{R}_{bb}(\tau) = \sigma_b^2 e^{-\beta|\tau|} . \cos(\alpha \,|\, \tau \,|) \,, \qquad (15)$$

where β and α are positive quantities, have the same dimension (1/time) and their values are chosen to fit an empirical ACS of the actual process experimental data. In contrast with the ACFs of GM processes that assume positive values only, the ACF of a PR process assumes negative values as well, which makes it a more general ACF that can correspond to a broader class of random variables (Andreyev 1969). Similarly as for GM processes, ACFs are generated for PR processes using Equ. (15) assuming 8 hours of data and variable values for β and α. These ACFs are shown in Fig. 5.

However, it should be mentioned here that a special PR process occurs when considering the particular case of a bandlimited WN process. The ACF of such process is given in Brown and Hwang (1997) as:

$$\mathbf{R}_{bb}(\tau) = 2WS_{bb} \frac{\sin(2\pi W \tau)}{2\pi W \tau} \,, \qquad (16)$$

where W is the physical bandwidth of the process.

Since the PR process is defined by two parameters (β and α), two state variables are needed to represent the process, and hence, a PR process can be considered as a 2nd order process. Moreover, the ACF formulae of both PR and GM processes involve an exponential. Therefore, sometimes a 2nd order GM process is generalized by combining it with a PR process. One example of such 2nd order GM general ACF is given in Grewal and Andrews (2001). It is of the form:

$$\mathbf{R}_{bb}(\tau) = \frac{1}{\cos\alpha} \sigma_b^2 e^{-\beta|\tau|} \cos(\ \beta \,|\, \tau \,| - \alpha\ ) \,, \qquad (17)$$

where β and α are determined to fit a computed ACF of the actual process. A graphical representation of Equ. (17) with variable values of β and α is shown in Fig. 6. Compared to Fig. 4 and Fig. 5, Fig. 6 indicates clearly that the ACF of the generalized 2nd order GM process is a compromise between the ACFs of GM and PR processes.

3 ACS of Inertial Experimental Data

In the previous Section, the ACFs of a number of random processes have been shown. As indicated before, the sensor residual errors of most inertial systems are assumed to follow a 1st order GM process. To investigate

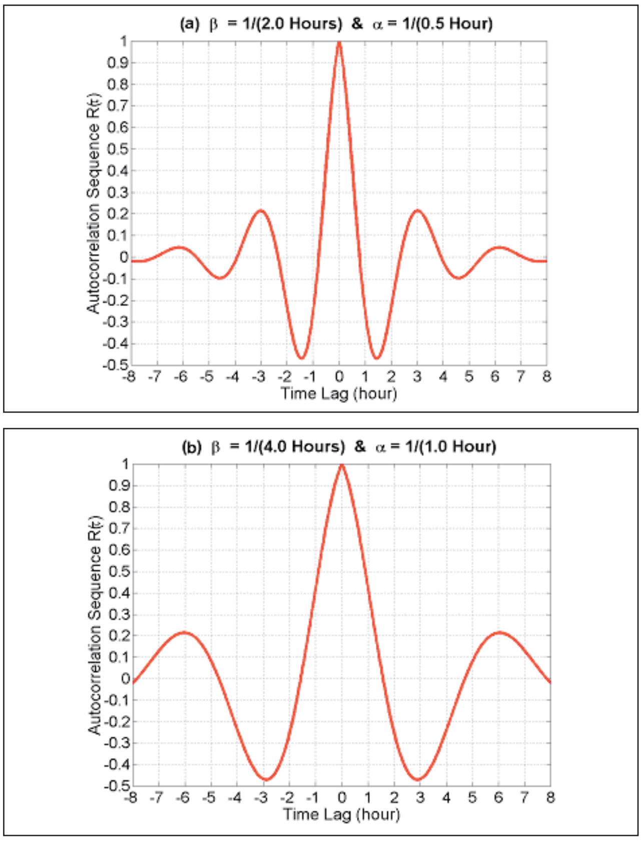

Fig. 5: The Generated ACFs for Different Periodic Random (PR) Processes

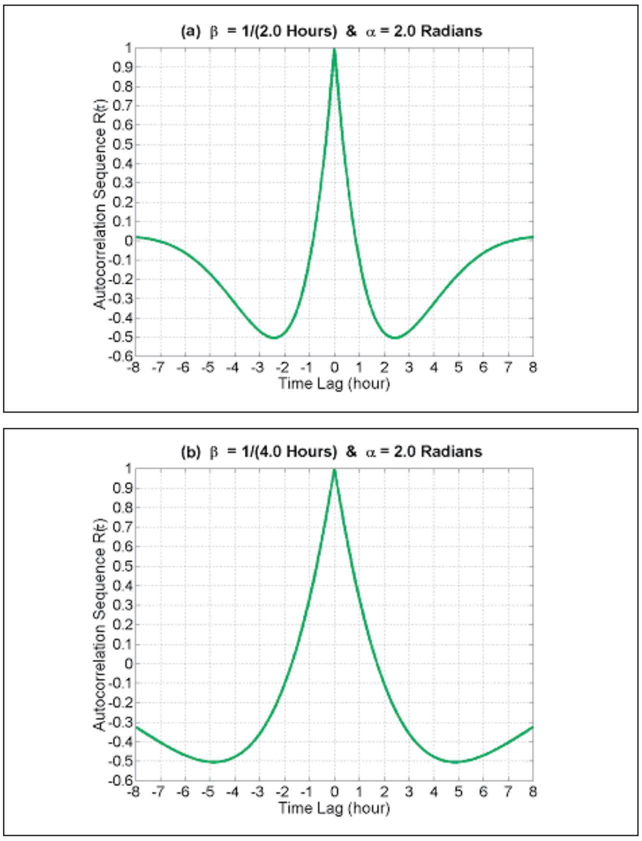

Fig. 6: The Generated ACFs for the Combined 2nd Order GM and PR Processes

the validity of such an assumption, or in other words, to determine the appropriate random process for modeling INS sensor errors, the ACSs of some INS measurements have been studied. Three Inertial Measuring Units (IMUs) are used for this purpose: a navigation-grade (high accuracy) IMU (Litton LTN 90-100 with a gyro drift of 0.01 deg/h), a high-end tactical-grade (medium accuracy) IMU (Honeywell HG1700 with a gyro drift of 1.0–10.0 deg/h) and a low-cost (low accuracy) IMU (Crossbow AHRS400CC-100 with a gyro drift of 200 deg/h). For each IMU, 8 hours of static data was collected. After subtracting the mean of the measurements for all sensors, the data was used for generating an ACS for each sensor. One sensor from each IMU (an accelerometer) is chosen to illustrate the obtained ACSs. For the rest of the sensors, similar ACSs were obtained.

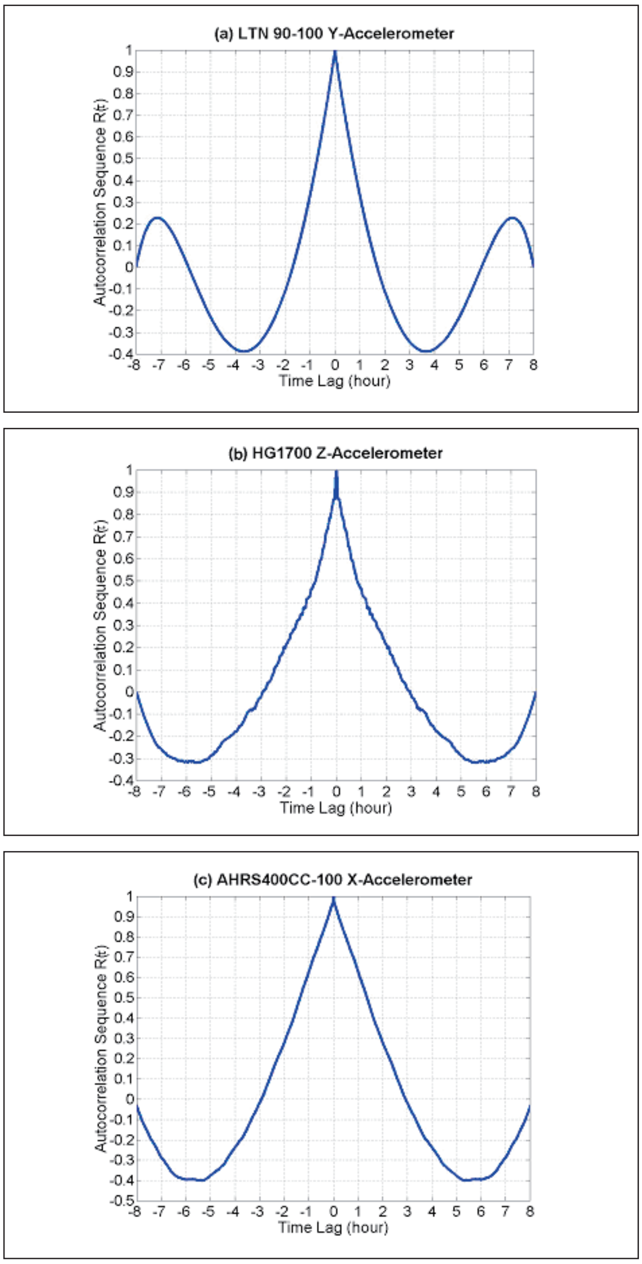

Fig. 7: The Computed ACSs for High, Medium and Low Accuracy Inertial Sensors

Figs. 7a–7c show these computed ACSs. Fig. 7 indicates clearly that a 1st order GM process may not be adequate in all cases to model inertial sensor errors. The shape of the ACS is often different from that of a 1st order GM process (Fig. 3).

By inspecting Fig. 7, it appears that most of the computed ACSs fall into the category of higher-order generalized GM processes or PR processes. As mentioned before, the required parameters for GM or PR process models (β and/or α) are determined based on the actual experimental data, i. e. by fitting an empirical ACS. However, Fig. 7 shows that the determination of an accurate ACS from experimental data is rarely done due to the fact that the data collected is limited and finite. In turn, the obtained values for β or α will change with the change in data length used for computing the corresponding ACS.

4 ACS Accuracy of Inertial Experimental Data

A more serious problem than the numerical difficulties is, however, a theoretical problem pointed out by Bendat and Piersol (1971) and further discussed by Brown and Hwang (1997). For a Gaussian zero-mean random process, the following relation is satisfied:

$$\sigma^2_{\mathbf{R}_{bb}(\tau)} \approx \frac{4}{T}\int_0^{-\infty} \bar{\mathbf{R}}^2_{bb}(\tau)\,d\tau\,, \tag{18}$$

where:

$\sigma^2_{\mathbf{R}_{bb}(\tau)}$ ...... is the variance of $\mathbf{R}_{bb}(\tau)$ (the ACF determined from a finite record of experimental data, i. e. the sample ACF).
T ...... is the time length of experimental data.
$\bar{\mathbf{R}}_{bb}(\tau)$ ...... is the theoretical ACF of the process (i. e. the ensemble ACF).

Equ. (18) can be used to get a »rough« estimate of the needed amount of data to reach a certain desired accuracy (uncertainty level) of $\mathbf{R}_{bb}(\tau)$. Obviously, Equ. (18) is valuable only if $\bar{\mathbf{R}}_{bb}(\tau)$ is known. Therefore, to illustrate the analysis, a 1st order GM process is assumed [$\bar{\mathbf{R}}_{bb}(\tau)=\sigma_b^2 e^{-\beta_1|\tau|}$]. Substituting this $\bar{\mathbf{R}}_{bb}(\tau)$ in Equ. (18) yields:

$$\begin{aligned}\sigma^2_{\mathbf{R}_{bb}(\tau)} &\approx \frac{4}{T}\int_0^{-\infty} \bar{\mathbf{R}}^2_{bb}(\tau)\,d\tau \approx \frac{4}{T}\int_0^{-\infty} \sigma_b^4 e^{-2\beta_1|\tau|}\,d\tau \\ &\approx \frac{2\sigma_b^4}{T\beta_1} \approx \frac{2\sigma_b^4\tau_{c_1}}{T}\,.\end{aligned} \tag{19a}$$

The accuracy (or uncertainty level) of $\mathbf{R}_{bb}(\tau)$ is defined as the ratio of the standard deviation of $\mathbf{R}_{bb}(\tau)$ (i. e. $\sigma_{\mathbf{R}_{bb}(\tau)}$) to the variance of the process (i. e. σ_b^2). By rearranging Equ. (19a) and taking into account the above definition of accuracy, we get:

$$\sigma^2_{\mathbf{R}_{bb}(\tau)} \approx \frac{2\sigma_b^4 \tau_{c_1}}{T}$$
$$\Rightarrow \quad \frac{\sigma^2_{\mathbf{R}_{bb}(\tau)}}{\sigma_b^4} = (\text{accuracy})^2 \approx \frac{2\tau_{c_1}}{T}. \quad (19b)$$

Therefore, if the desired uncertainty level is 10 % for example, the required time length T of experimental data will approximately equal $2\tau_{c_1}/(\text{accuracy})^2 \approx 2\tau_{c_1}/(0.10)^2 \approx 200\tau_{c_1}$, i. e. 200 times the correlation time of the process. Assuming a reasonable correlation time of 1.0 hour, this means that 200 hours of data is required for estimating the ACS with 10 % accuracy. Taking into account the high data rate of INS sensors (50–400 Hz), it is unlikely that this requirement will be used in any practical work. The above analysis can be also performed for PR processes and GM processes of any order and it will lead to the same conclusion.

On the other hand, Equ. (18) can be used to give an approximate estimation of the accuracy of the ACS obtained from experimental data of known finite length T. In this case, $\bar{\mathbf{R}}_{bb}(\tau)$ is assumed to be known and the process parameters (β and/or α) are estimated from the obtained ACS $\mathbf{R}_{bb}(\tau)$. To estimate the accuracy of the obtained ACSs in Fig. 7, a 1st order GM $\bar{\mathbf{R}}_{bb}(\tau)$ is assumed again. In this case, Figs. 7a–7c show that the estimated correlation times are: 5/6, 4/3 and 7/4 hours, respectively. The accuracy of these ACFs is computed using Equ. (19b) as: accuracy $\approx \sqrt{2\tau_{c_1}/T} \approx \sqrt{2\tau_{c_1}/8} \approx 0.5\sqrt{\tau_{c_1}}$. Substituting the above estimated correlation times, the approximate accuracy of the obtained ACSs are: 46 %, 58 % and 66 %, respectively. These numbers indicate that it is very difficult to obtain an accurate ACS from inertial experimental data. If higher-order GM processes were assumed instead, the computed ACS accuracies will be even worse. Therefore, it is unlikely that the INS sensor errors can be accurately estimated by using the parameters of an ACS that has been determined from actual data. Hence, other methods rather than computing the ACS should be investigated. This will be discussed in the following Section.

5 Autoregressive (AR) Processes

To avoid the problem of inaccurate modeling of inertial sensor errors due to inaccurate ACS determination, another method is introduced in this Section. The method, known as Autoregressive (AR) process modeling, has been introduced almost 30 years ago but it has not been used before for modeling errors of all INS sensors inside the KF navigation error model. Compared to the other random processes discussed in this paper, AR processes have more modeling flexibility since they are not always restricted to only one or two parameters. In many applications with quantities that involve time series of measurements, AR processes are used to model (estimate) the stochastic part of such quantities (Box and Jenkins 1976, Granger and Andersen 1978, Young 1984, Klees and Broersen 2002). Inertial data is a time series of measurements that contain both systematic and stochastic error parts. Therefore, AR models will be used in this Section to describe the INS sensor stochastic errors. Based on the obtained ACSs in Fig. 7, it has been decided to model the randomness of the inertial measurements in this Section using an AR process of order higher than one. In the time domain, the AR process is written as:

$$y(k) = -\sum_{n=1}^{p} \alpha_n y(k-n) + \beta_0 x(k), \quad (20a)$$

i. e.

$$y(k) = -\alpha_1 y(k-1) - \alpha_2 y(k-2) - \ldots\ldots - \alpha_p y(k-p) + \beta_0 x(k). \quad (20b)$$

To apply AR models, and in analogy with the discussed shaping filters, the input to the AR model x(k) will be a sequence of zero-mean white sequence w_k while the output y(k) will be the inertial sensor residual bias b_k. The problem in this case is to determine the AR model parameters (predictor coefficients) α_n. This is performed by minimizing the prediction error e(k) between the original signal y(k) represented by the »AR process« of Equ. (20) and the estimated signal $\hat{y}(k)$, which is estimated by an »AR model« of the form:

$$\hat{y}(k) = -\sum_{n=1}^{p} \alpha_n y(k-n). \quad (21)$$

The cost function for this minimization problem is the sum of squared errors $\mathcal{E}(k)$ of e(k), sometimes called the energy of e(k). Several methods have been reported to estimate the α_n parameter values by fitting an AR model to the input data. In Nassar et al. (2003), three different methods (Yule-Walker, Covariance and Burg's) were investigated. The results showed that Burg's method gives the minimum estimation mean square error and hence this method will be the one to be used in this paper. In addition, it has been shown in Nassar et al. (2003) that due to the high level of existing noise in all inertial measurements, de-noising of inertial sensor data using wavelet analysis is crucial for an accurate determination of the AR model parameters. Therefore, all AR parameters in the paper as well as all plotted ACSs in Section 3 were estimated using de-noised data.

To show the analysis of AR model parameter determination, one set of the three static data used before for

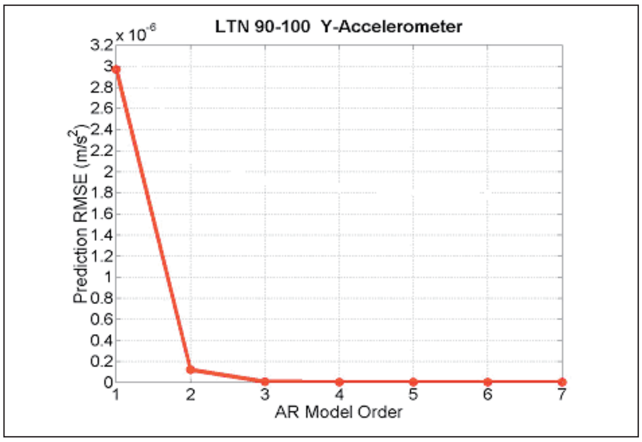

Fig. 8: AR Model Prediction RMSE

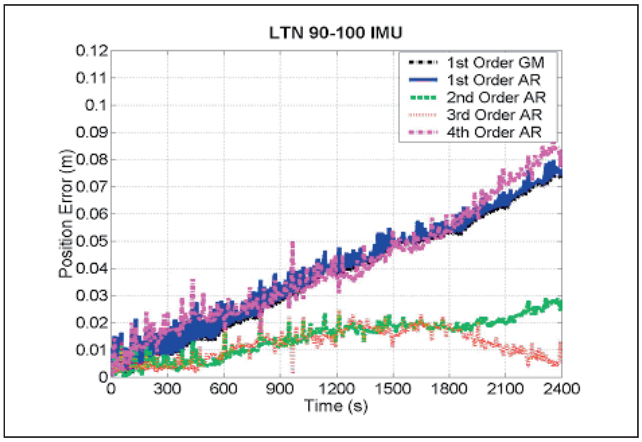

Fig. 9: LTN 90-100 Position Errors Using Different Stochastic Sensor Error Models

computing the ACSs in Fig. 7 is utilized. The chosen set is the LTN 90-100 IMU data, however, similar results were obtained for the other two IMUs. Also, one sensor was selected as an example (y-accelerometer). The AR model parameters were then estimated as well as the corresponding prediction Root Mean Square Error (RMSE) for all sensors using Burg's method. Different AR model orders are used in the analysis. The other sensors gave similar results. The y-accelerometer prediction RMSE using Burg's method with different AR model orders (1 to 7) is plotted in Fig. 8. The figure indicates that the RMSE is close to its minimum value after applying only a 3rd or 4th order AR model. This is very important from a numerical point of view since the addition of the corresponding INS sensor error states in this case into the used KF algorithm will not affect its stability.

Tab. 2: Position Errors Using Different Stochastic Processes for Modeling Sensor Errors

Sensor Err. Modeling	LN 90-100 IMU			HG1700 IMU		
	Errors (m)			Errors (m)		
	Mean	Max	RMS	Mean	Max	RMS
WN	0.89	1.57	0.97	0.91	1.51	0.96
RC	0.09	0.18	0.10	0.42	0.88	0.44
RW	0.04	0.08	0.04	0.42	0.88	0.44
1st Ord. GM	0.04	0.08	0.04	0.42	0.88	0.44
1st Ord. AR	0.04	0.08	0.04	0.42	0.88	0.44
2nd Ord. AR	0.01	0.03	0.02	0.23	0.91	0.26
3rd Ord. AR	0.01	0.03	0.01	0.23	0.67	0.26
4th Ord. AR	0.04	0.09	0.05	0.41	1.09	0.45

6 Results

To test the accuracy of the different random processes discussed in Section 2 and the efficiency of the suggested AR processes in modeling inertial sensor errors, another one hour of static data (rather than the 8 hours data used before) was collected by both the LTN 90-100 and the HG1700 IMUs. The first 20 minutes of each data set were used for alignment while the last 40 minutes were used for testing. For each IMU, the data (without any de-noising) was processed with using ZUPTs as updates for the KF. Sensor errors are modeled first by one of the random processes: WN, RC, RW as well as by the commonly used 1st order GM process. Then, these errors are modeled by AR processes of different orders (1 to 4). Position errors are then computed for each model. The statistical parameters of the LTN 90-100 and HG1700 position errors obtained from each model are given in Tab. 2. In addition, Fig. 9 shows the LTN 90-100 position errors using 1st order GM model and AR models of different orders.

From Tab. 2, it is clear that a WN process is not adequate for modeling inertial sensor biases of both IMUs. This fact agrees with the obtained ACSs of Fig. 7, which indicated a correlation between residual biases. For the LTN 90-100, Tab. 2 shows that a RC process is not adequate either. Moreover, for both IMUs, RW and 1st order GM processes provide similar results. However, this can be explained by comparing the coefficients of b_k in Equ. (9) of a RW and Equ. (13) of a 1st Order GM and taking into account that the correlation time of the 1st order GM process is fairly large and the inertial data has a high data rate (64 Hz). Therefore, the term $(1-\beta_1\Delta t)$ of Equ. (13) will be very close to 1.0, which is equivalent in this case to a RW process. For the HG1700, the RC process gave similar results to RW and 1st order GM processes. In this case, and since these three processes are 1st order shaping filters, this means that the addition of a driving white sequence for 1st order random processes does not have a major effect for the HG1700 IMU.

For both the LTN 90-100 and HG1700 IMUs, 1st order GM and 1st order AR models provide the same numerical results. This is expected since both models are of the same order. Compared to the 1st order GM and AR model results, the LTN 90-100 position errors are improved by 64 % and 70 %, respectively, after applying AR models of 2nd and 3rd orders. In case of the HG1700, the improvement is 40 % and 42 %, respectively. This indicates the efficiency of the AR models of orders higher than one. However, the worst AR results are obtained from the 4th order AR model. This could be the result of two possible causes. The first one is that the KF starts to diverge due to the instability and model complexity resulting from adding more error states. The second cause is that the 4th order AR model does not decrease the prediction RMSE obtained from the 3rd order AR model. This could result in an over-parameterization of the model introducing oscillating features into the solution.

7 Stability of Stochastic Model Parameters

In the previous Section, all obtained results showed that the performance of AR processes of an appropriate order is better than the performance of all other implemented processes, including the most widely used 1st order GM process. However, AR processes and the other implemented random processes have the common problem that the process model coefficients (parameters) are estimated from experimental data. In Section 2, it has been addressed that the obtained values of the parameters of the other random processes (especially GM processes) will change with the change in data length used for their computation. Therefore, the question arises if this is also true for the estimated parameters of AR processes.

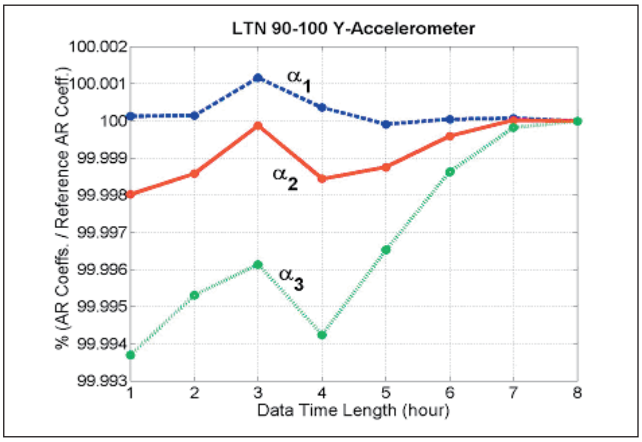

Fig. 10: Variation of the 3rd Order AR Model Parameters

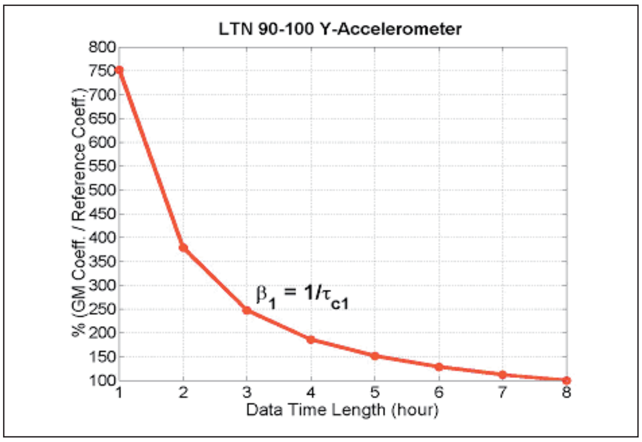

Fig. 11: Variation of the 1st Order GM Model Parameter

To answer this question, the AR model parameters should be computed using different data time lengths. For this purpose, the measurements of one sensor of the three data sets used in Section 2 are chosen. The selected sensor measurements are the 8-hour data span of the LTN 90-100 y-accelerometer. All other sensors show similar results. For the analysis, a 3rd order AR model is assumed, and hence 3 coefficients (α_1, α_2 and α_3) are estimated for different time lengths of the data (1, 2, 3, 4, 5, 6, 7 and 8 hours). Therefore, for each coefficient, 8 values are computed. To check the stability of the AR model coefficients, the computed 8 values of each coefficient are compared to a reference value of such coefficient. The reference value here is the one used in the analysis performed in the previous Section, i. e. the value that corresponds to 8 hours. The comparison is performed by obtaining the percentage resulting from dividing the 8 values of each coefficient by its reference value. The results of such analyses are shown in Fig. 10.

Fig. 10 indicates that the variations between the values of each AR model parameter, obtained using different data time lengths, are very small. The maximum variation occurs in α_3 with an amount of 0.0062 %, which is obviously negligible. Moreover, Fig. 11 shows for all coefficients (α_1, α_2 and α_3) that their values start to converge after using 5 hours of data and are almost constant after using 7 hours of data. This fact is very important since it confirms that there is no need to use larger data sets for computing the AR model parameters. Finally, to assess that this is not the case for other random processes discussed in Section 2, the same analysis is performed using the same sensor data and assuming a 1st order GM process.

As shown in Section 2, the 1st order GM model parameter β_1 is obtained from the computed ACS. To investigate the variation of β_1 in this case, the ACS is computed using different time lengths of data (i. e. 1, 2, 3, 4, 5, 6, 7 and 8 hours, respectively), and then β_1 is obtained for each time length. Similar to the AR model analysis, the comparison between the 8 values of β_1 is performed by dividing each value by the value of β_1 obtained using the 8 hours data (the reference value). The resultant percentages are shown in Fig. 11. Compared to Fig. 10, Fig. 11 depicts that the variation level of β_1 is very large and more data is needed to reach the convergence level. Again, this agrees with the results obtained before in Section 2.

8 Conclusions

The overall objective of this article was to improve the accuracy obtained in modeling inertial sensor errors. Detailed analyses of different stochastic processes have been investigated and implemented. In the paper, it has been shown that the parameters of any random process that are estimated based on an actual Autocorrelation Sequence (ACS) are changing with the data length. Therefore, it is not possible to estimate the inertial sensor errors accurately using the parameters of such an ACS. In addition, when studying the ACS of inertial data, it appears that the frequently used 1st order Gauss-Markov (GM) process is not always adequate for modeling inertial sensor errors. Compared to a 1st order GM model, the obtained INS position errors using the suggested Autoregressive (AR) models of 2nd and 3rd orders were better by 40 % to 70 %. Finally, the analysis showed that the variation of the AR model parameters is very small compared to the corresponding parameters of GM processes that are estimated from an actual ACS of the same data, which confirms the stability of the AR model parameters.

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