

State Monitoring of an Airborne Sensor

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Abstract

This short paper describes how to monitor the flight state of the interested sensor via GPS aircraft state monitoring. The theoretical relationships are outlined in detail and a general algorithm to compute flight state angles is described. An example is given to demonstrate the application of the theory. In aerogravimetry such correction between GPS antennas and aerogravimeter is necessary and should be included in standard data processing algorithm in the future.

Zusammenfassung

In diesem Artikel wird beschrieben, wie sich die Position und Geschwindigkeit eines Sensors auf einer Flugplattform mittels GPS adäquat überwachen lässt. Die theoretischen Zusammenhänge werden im Einzelnen diskutiert und ein allgemeiner Algorithmus zur Berechnung der verschiedenen Winkel wird beschrieben. An einem Beispiel wird die praktische Anwendung der Theorie gezeigt. In der Aerogravimetrie ist eine derartige Korrektur zwischen den GPS-Antennen und dem Fluggravimeter erforderlich und sollte daher zukünftig Bestandteil der Standard-Prozessierung der Daten sein.

1 Positioning from GPS Antennas to the Interested Sensor

The location of the interested sensor of a remote sensing system usually differs from GPS antennas mounted on the aircraft for flight state monitoring. Therefore the velocity and acceleration of the sensor are usually different from that of GPS antennas. For precise application, it is necessary to determine the flight state of the interested sensor from the flight state of the aircraft.

Flight-state of an aircraft can be monitored by using several GPS antennas fixed on the outside of the aircraft. Flight-state is usually represented by so-called »state angles« (heading, pitch, and roll). They are rotation angles between the body and the local horizontal coordinate frames of the aircraft. The axes of the body frame are selected as follows: the x^b axis points out the nose, the y^b axis points to the right parallel to the wing, and the z^b axis points out the belly to form a right-handed coordinate system, where b denotes the body frame. The axes of the horizontal frame are defined as follows: the x^h axis points to the north, the y^h axis points to the east, and the z^h axis points to the vertical direction to form a left-handed coordinate system, where h denotes the horizontal frame. The body frame can be rotated to be aligned

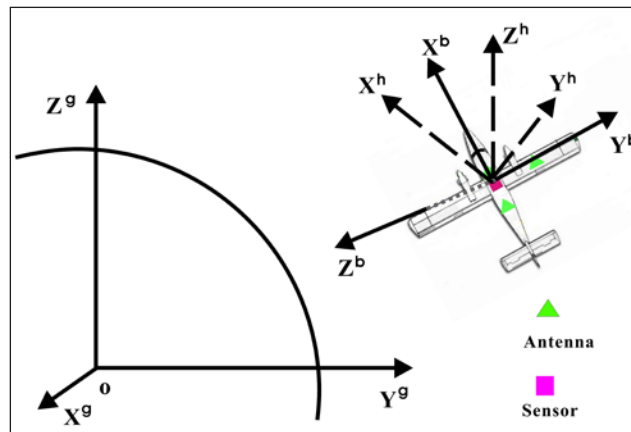


Fig. 1: Body-fixed frame and body horizontal frame in global frame

to the local horizontal frame in a positive, right-handed sense, which is performed in three steps and changes the direction of the third axis. First, the body frame is rotated about the local vertical downward axis z^b by angle ψ (heading). Next the body frame is rotated about the new y^b axis by angle θ (pitch). Then the body frame is rotated about the new x^b axis by angle ϕ (roll). Finally, change the direction of the axis z^b . In the local horizontal coordinate system the heading is the azimuth of axis x^b of the body frame, the pitch is the elevation of axis x^b of the aircraft and the roll is the elevation of axis y^b of the aircraft. Note that the directions of the axis x^b and the velocity vector of aircraft are usually not the same. Through kinematic positioning, three flight state monitoring angles can be computed (Cohen 1996, Xu 2003, 2007). However, the derivations are under simplified assumptions and the formulae are not generally valid for general cases.

GPS is used to determine the position and velocity of antennas. But one usually needs the position and velocity (as well as acceleration) of the interested sensor (e.g., aerogravimeter). Formulae to compute or to transform the monitoring position and velocity to the position and velocity of the sensor are well-known. The algorithm, however, to compute the flight-state angles is complicated. This paper will describe an easier way of flight-state monitoring to derive the position and velocity (as well as acceleration) of the sensor. This is significant for kinematic platform monitoring practice in many cases. The relationship between global and horizontal as well as body-fixed frames is shown in Figure 1.

Procedure: Define the geometric center point of the three antennas by

$$X^b(c) = \frac{1}{3}(X^b(1) + X^b(2) + X^b(3)) \quad (1)$$

and translate the origin of the body frame to the center point by

$$X^b(1) - X^b(c), X^b(2) - X^b(c), X^b(3) - X^b(c), \quad (2)$$

where $X^b(1), X^b(2), X^b(3)$ are three coordinates of the antennas in body frame. It is obvious that the origin point may be defined alternatively. According to the definition of the body frame and horizontal coordinate system as well as discussions above, one has (cf. Xu 2007)

$$X^h(i) = R_4 R_1(\phi) R_2(\theta) R_3(\psi) X^{db}(i), \quad i = 1, 2, 3, \quad (3)$$

where $X^h(i)$ are coordinate vectors in the local horizontal frame and $X^{db}(i) = X^b(i) - X^b(c)$; R_i is the rotational matrix around the i -th axis ($i=1,2,3$); R_4 is the function of the sign change of the third axis. There is

$$R = R_4 R_1(\phi) R_2(\theta) R_3(\psi) = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix},$$

$$= \begin{pmatrix} \cos\theta \cos\psi & \cos\theta \sin\psi & -\sin\theta \\ \sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi & \sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi & \sin\phi \cos\theta \\ -\cos\phi \sin\theta \sin\psi + \sin\phi \cos\psi & -\cos\phi \sin\theta \sin\psi + \sin\phi \cos\psi & -\cos\phi \cos\theta \end{pmatrix}. \quad (4)$$

The coordinate vectors of the three GPS antennas and the sensor ($X^b(1), X^b(2), X^b(3), X^b(4)$) are well known by the body coordinate system definition and measurement after the instrumental montage. The coordinates of the three antennas in the global coordinate system (e.g. ITRF2000) are known via GPS positioning and are denoted by ($X^g(1), X^g(2), X^g(3)$). All coordinate vectors have three components of x, y, z . The geometric center of the three antennas in the global GPS frame is

$$X^g(c) = \frac{1}{3}(X^g(1) + X^g(2) + X^g(3)). \quad (5)$$

Using that geometric center point as origin a local horizontal frame can be defined and the three known GPS positions $X^g(1), X^g(2), X^g(3)$ can be transformed to the local horizontal frame by (cf. Xu 2007)

$$X^h(i) = R_0 X^g(i), \quad i = 1, 2, 3, \quad (6)$$

$$R_0 = \begin{pmatrix} -\sin\varphi \cos\lambda & -\sin\varphi \sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi \cos\lambda & \cos\varphi \sin\lambda & \sin\varphi \end{pmatrix}, \quad (7)$$

where φ and λ are the geodetic latitude and longitude of the geometric center point of the three antennas in the global GPS frame respectively.

Then the flight state monitoring angles can be determined by Eq. (3). There are nine equations and three angular variables. Because the three angular unknowns are arguments of sine and cosine functions and multiplied to each other, the problem can not be solved straight forward. However, physically a unique set of solutions is at hand and has been solved in different ways by several authors many years ago (cf. e.g. Sanso 1973, 1976). Alternatively, the flight state angles can be numerically searched for. An algebraic method can be outlined as follows: Eq. (3) for $i=1,2,3$ can be used to form three equation systems of unknown (R_{i1}, R_{i2}, R_{i3}). Each of them have a unique solution vector and therefore one has the solution of all elements of the matrix R . Comparing the elements of R with formulas given in Eq. (4) the three flight state angles can be determined. This can be realised by following steps: from the value of R_{13} the pitch angle θ (elevation of axis x^b of the aircraft) can be uniquely determined (in non-military flight). Then from the values of R_{11} and R_{12} the heading angle ψ (azimuth of axis x^b) can be determined. Values of R_{23} and R_{33} can be used to determine the roll angle ϕ (elevation of axis y^b of the aircraft). This is a very simple algorithm to determine the flight state angles compared with existing algorithms.

After the flight state angles are determined the coordinates of the interested point, e.g., the gravimeter, can be computed by

$$X^h(4) = R_4 R_1(\phi) R_2(\theta) R_3(\psi) (X^b(4) - X^b(c)), \quad (8)$$

where $X^b(4)$ is the coordinate vector of the interested point in the body frame and $X^h(4)$ is the coordinate vector of the interested point in the local horizontal frame. $X^h(4)$ can then be transformed to the global GPS frame. In this way, the coordinate vector of the interested point in the global GPS frame can be obtained. Furthermore, the velocity of the interested point can be obtained by numerical differentiation.

2 Velocity Transformation from GPS Antennas to the Interested Sensor

Velocities of the three GPS antennas can be determined by using Doppler observations. To get the velocity of the interested sensor (e.g. aerogravimeter), the problem can

be outlined as follows. In a fixed body with known positions and velocities of three points, search for velocity of a known point in the body. The problem turns out to be geometric. One has three independent distance relations of

$$\begin{aligned} (x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2 &= d_{41}^2, \\ (x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2 &= d_{42}^2, \\ (x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2 &= d_{43}^2, \end{aligned} \tag{9}$$

where the indices 1, 2, 3, 4 are used to identify the number of points. The distance between point i and j is represented by d_{ij} . Differentiate Eq. (9) with respect to time t , one has

$$\begin{aligned} (x_4 - x_1)(\dot{x}_4 - \dot{x}_1) + (y_4 - y_1)(\dot{y}_4 - \dot{y}_1) \\ + (z_4 - z_1)(\dot{z}_4 - \dot{z}_1) &= 0, \\ (x_4 - x_2)(\dot{x}_4 - \dot{x}_2) + (y_4 - y_2)(\dot{y}_4 - \dot{y}_2) \\ + (z_4 - z_2)(\dot{z}_4 - \dot{z}_2) &= 0, \\ (x_4 - x_3)(\dot{x}_4 - \dot{x}_3) + (y_4 - y_3)(\dot{y}_4 - \dot{y}_3) \\ + (z_4 - z_3)(\dot{z}_4 - \dot{z}_3) &= 0. \end{aligned} \tag{10}$$

There are three equations with three unknowns of velocity components and a unique set of solutions. The velocity of the interested sensor can be determined this way.

The acceleration of the sensor can be obtained by numerical differentiation of the velocity series.

3 Numerical Example and Conclusion

Data of the NorthGRACE campaign on 10th June 2007 are used. The flight trace and height profile are shown in Figure 2 and 3 respectively. Height differences between the antenna and the aerogravimeter are given in Figure 4. The acceleration differences of the vertical component between the antenna and the aerogravimeter are shown in Figure 5. It is obvious that the height differences will not affect the results considerably, because they are within the requirement of the height precision (less than one meter). However, the acceleration differences are not very small (as shown in Figure 5, the amount could reach up to 1000 mgal). After filtering, the amounts of the acceleration differences are greatly reduced (cf. Figure 6, the maximum during the flight (not including the times

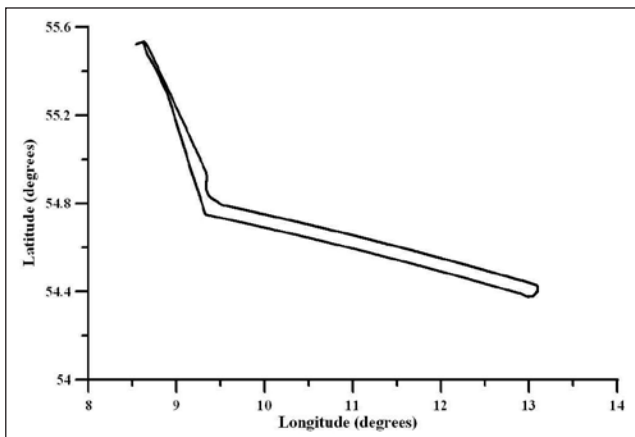


Fig. 2: Flight trace in longitude-latitude plane

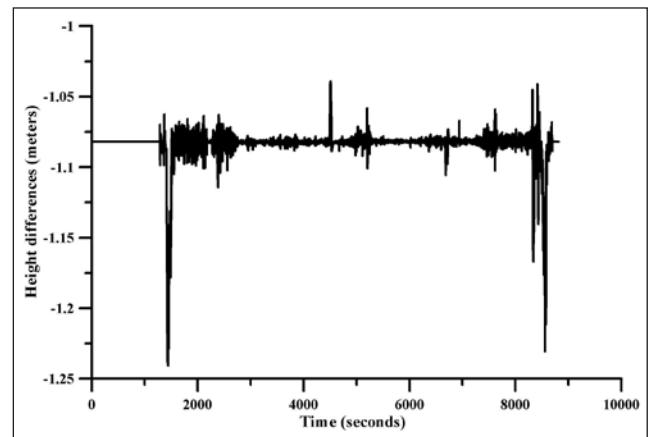


Fig. 4: Height differences between antenna and aerogravimeter

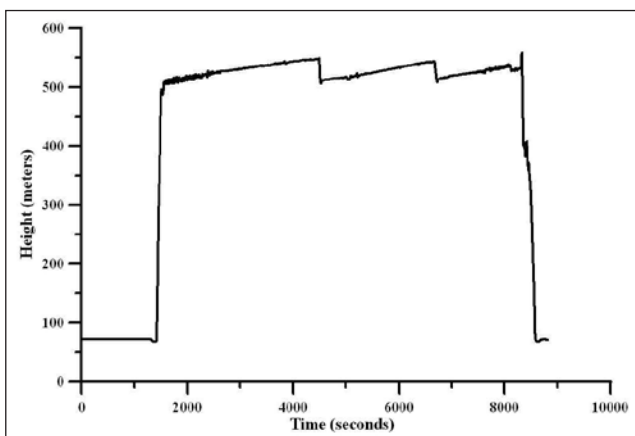


Fig. 3: Height profile of the flight trace

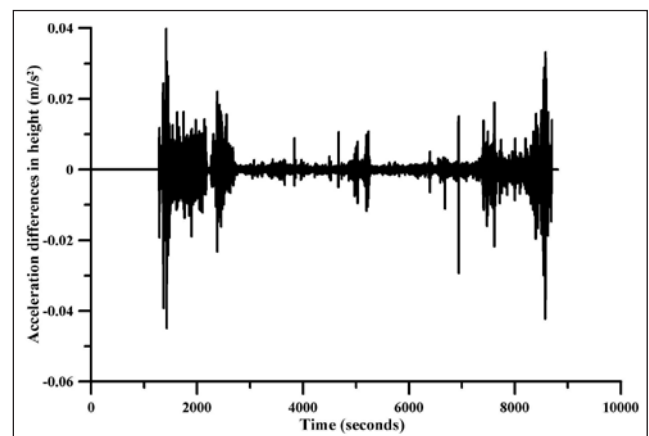


Fig. 5: Acceleration differences between antenna and aerogravimeter in vertical component

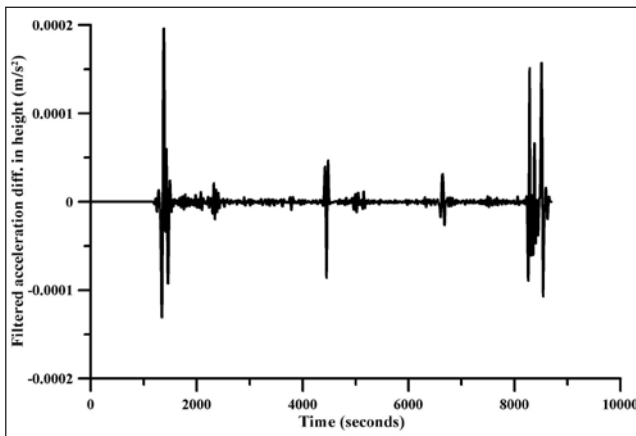


Fig. 6: Filtered acceleration differences between antenna and aerogravimeter in vertical component

of take off and landing as well as turning around) could reach up to 2.5 mgal). This indicates that the relations (or corrections) between the antennas and the aerogravimeter should be taken into account in the standard algorithm of data processing in the future for these cases, the accuracy requirement is better than 2.5 mgal.

To monitor the flight state of interested sensor usually three GPS antennas are needed. This could be a problem in many practical cases. When only one GPS antenna is used, the flight state can be monitored approximately by using antenna positions of two adjacent epochs to compute the heading and pitch and let the roll as zero (this is nearly true in the most cases of aerogravimetry).

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