Two Earth Gravity Models from the Analysis of Global Crustal Data

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Summary

Based on the standard scheme of forward gravity field modeling, the density and stratification information contained in the recently released global database CRUST 2.0 is used for the computation of two independent Earth gravity models. Using a standard Airy/Heiskanen isostatic hypothesis and a radially distributed compensation mechanism, the two models are complete only up to degree and order 90, due to the given resolution of the crustal data. The newly derived fields are compared with the EGM96 observed field in terms of geoidal heights for selected bandwidths. As the CRUST 2.0 data emerged from observed seismic and compiled tectonic information, the new topographic/isostatic (t/i) models prove useful especially to local and regional gravity field investigations.

Zusammenfassung

Zwei unabhängige Erdschwerefeldmodelle werden auf der Basis der Dichte- und Schichteninformation der globalen Krustedatenbank CRUST 2.0 hergeleitet. Die Berechnung erfolgt nach dem bekannten Schema der direkten Schwerefeldmodellierung, mit Anwendung einer isostatischen Hypothese nach Airy/Heiskanen sowie eines radial verteilten Kompensationsmechanismus. Wegen der gegebenen Auflösung der Krustedatenbank sind die zwei Modelle nur bis zu Grad und Ordnung 90 vollständig. Die neuen Modelle werden mit dem EGM96 Feld anhand der entsprechenden Geoidundulationen für ausgewählte Wellenlängen verglichen. Da die CRUST 2.0 Daten aus beobachteten seismischen Daten und bekannten tektonischen Information kompiliert wurden, bieten die neuen topographisch-isostatischen Modelle eine nützliche Referenzlösung für lokale und regionale Schwerefelduntersuchungen.

1 Introduction

The recovery of the gravity field related information, which is contained in global databases describing the Earth's structure and consistency, is one of the central topics of modern geodetic research. This interest is expressed, not lastly, by the recent adaptation from the International Association of Geodesy of the Special Study Group SG2.2 on »Forward Gravity Field Modeling Using Global Databases« under its Commission 2 »Gravity Field« (see http://www.ceegs.ohio-state.edu/iag-commission2/sg2.2.htm).

The above mentioned databases were – up to now – mostly elevation models, lately however, several global crustal models have been released including density and stratification information of the upper, middle and lower crust with ever growing resolution. The CRUST 5.1 model was one of the first global databases providing density and depth information of a total of eight layers of the lithospheric crust and the uppermost mantle with a resolution of 5°×5° (Mooney et al. 1998). CRUST 2.0 is an updated version of the CRUST 5.1 model, assigning crustal structures at a 2°×2° scale (for a description see for example Tsoulis 2003). Both crustal models emerged from a combined analysis of seismic data with a detailed compilation of published maps of ice and sediment thickness.

Why should the geodetic community have interest in such data? The answer can be found in the basic theoretical background of gravity field modeling: the expansion of the gravitational potential in spherical harmonics. The coefficients $\bar{C}_{lm}$ and $\bar{S}_{lm}$ enter this fundamental equation as the main field unknowns. Their determination is the main purpose of all current and planned dedicated satellite gravity field missions. Their knowledge equals to the knowledge of the Earth's shape, the Earth's gravity field and their temporal variations. Apart from the analysis of satellite data one may derive a set of potential harmonic coefficients, or equally a gravity model from the analysis of global databases describing the Earth's crust. In practice, the derivation of geopotential harmonic coefficients is based on discrete data given on a reference surface, e. g. a sphere. Depending on the nature of these data, one obtains a different spectrum of the field. If, for example, we process global topography, the deduced set of coefficients will be dominated by the high frequencies of the field. The spherical harmonic analysis of satellite-to-satellite tracking data in combination with discrete gravity measurements registered by an accelerometer on board of the spacecraft, which serves a detailed description of non-gravitational forces (principle of CHAMP and GRACE), would provide on the other hand a different »fingerprint« of the gravity field, describing mainly its low-frequency, or equally its medium to long wavelength part of its spectrum. Shorter wavelength structures are expected to be recovered in the frame of the GOCE mission, whose principle is satellite gradiometry (ESA 1999). Thus, the harmonic analysis of global databases as the one presented in this paper is an indispensable part of the current gravity field research, as the respective spectra may act complementary to the satellite only models, especially in the frame of defining a combined reference Earth gravity model, or equally constructing so called synthetic Earth models (Claessens 2003).

2 Theoretical development

The gravitational potential at an arbitrary point in space P due to the Earth's density distribution is given by Newton's law

$$V_P = G\iiint_\Sigma \frac{\rho(Q)}{l_{PQ}}\,\mathrm{d}\Sigma_Q \tag{1}$$

with G the gravitational constant, $\rho(Q)$ the unknown density function and l_{PQ} the distance between P and an infinitesimal volume element $\mathrm{d}\Sigma Q$ at Q. Presuming P is situated outside or on a spherical shell including all attracting masses, the so-called Brillouin sphere (see Fig. 1), one gets for the spherical harmonic expansion of the inverse distance function $1/l_{PQ}$ the following expression

$$\frac{1}{l_{PQ}} = \frac{1}{r_P}\sum_{l=0}^{\infty}\left(\frac{r_Q}{r_P}\right)^l P_l(\cos\psi_{PQ}) \tag{2}$$

where r_Q and r_P denote the distances of Q and P from the origin in a spherical coordinate system, $r_Q < r_P$, $P_l(\cos\psi_{PQ})$ are the Legendre polynomials of degree l and ψ_{PQ} the angle linking attracting point Q to computation point P. Making use of the addition theorem of the spherical harmonic functions (Lense 1954) and introducing the abbreviation

$$Y_{lm}^{\alpha}(P) = \bar{P}_{lm}(\cos\theta_P)\begin{cases}\cos m\lambda_P & \text{for } \alpha = 0\\ \sin m\lambda_P & \text{for } \alpha = 1\end{cases} \tag{3}$$

with $\bar{P}_{lm}$ the fully normalized associated Legendre functions and m denoting the order, equation (1) becomes

$$V_P = \frac{GM}{\bar{R}}\sum_{l=0}^{\infty}\frac{\bar{R}^{l+1}}{r_P^{l+1}}\sum_{m=0}^{l}\sum_{\alpha=0}^{1} Y_{lm}^{\alpha}(P)C_{lm}^{\alpha}\ . \tag{4}$$

In this equation M represents a rough estimate of the Earth's mass using e.g. a mean density value of $\bar{\rho} = 5500\,\mathrm{kg\,m^{-3}}$, $\bar{R}$ denotes a mean Earth radius value ($\bar{R} = 6370\,\mathrm{km}$) and, after recalling the expression for the volume element in spherical coordinates $\mathrm{d}\Sigma_Q = r_Q^2\,\mathrm{d}r_Q\,\mathrm{d}\sigma_Q$, $\bar{C}_{lm}^{\alpha}$ stands for the dimensionless potential harmonic coefficients of the Earth's gravitational field

$$C_{lm}^{\alpha} = \begin{Bmatrix}\bar{C}_{lm}\\ \bar{S}_{lm}\end{Bmatrix}$$

$$= \frac{3}{2l+1}\frac{1}{\bar{\rho}\bar{R}^3}\frac{1}{4\pi}\iint_\sigma\left(\int_r \rho(Q)\left(\frac{r_Q}{\bar{R}}\right)^l r_Q^2\,\mathrm{d}r_Q\right)Y_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\ . \tag{5}$$

The retrieval of the coefficients $\bar{C}_{lm}^{\alpha}$ by processing either satellite only information (range and range-rate data of Low Earth Orbiting (LEO) satellites, satellite altimetry etc.), or by combining heterogeneous terrestrial (e.g. terrestrial, shipborne or airborne gravimetry) with satellite data, has been the objective of numerous gravity field models presented by many research groups over the last twenty years. The development of the joint NASA Goddard Space Flight Center and the American National Imagery and Mapping Agency (NIMA) geopotential model EGM96 (Lemoine et al. 1998) is a recent example of a combined gravity field model up to degree 360. Bouman (1997) offers an overview of the most extensively used models in geodesy and geodynamics. For an overview of the gravity models of the current and forthcoming satellite gravity field missions CHAMP, GRACE and GOCE see e.g. Reigber et al. (1996) and Balmino et al. (1998).

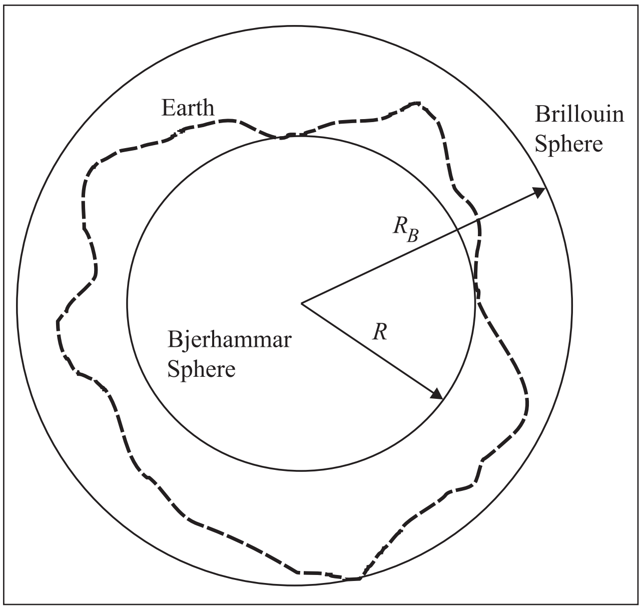

Fig. 1: Characteristic convergence areas in the expansion of the inverse distance function into spherical harmonics: the *Brillouin* and *Bjerhammar* spheres

Under circumstances the knowledge of the unknown behavior of the density function $\rho(Q)$ can provide an alternative path of computing $\bar{C}_{lm}^{\alpha}$, namely directly from equation (5). Unlike the inverse process of retrieving the spherical harmonic coefficients from satellite and/or gravity data based on equation (4), the computation of $\bar{C}_{lm}^{\alpha}$ using density knowledge according to equation (5), is a forward computation. However, the complete knowledge of the actual form of $\rho(Q)$ inside the Earth remains and will remain undetermined. What is feasible is either a global isostatic assumption according to a certain idealized theoretical model, or the assignment of observed local crustal structures to a global crustal model applying certain model restrictions. The former alternative using the Airy/Heiskanen isostatic model has been pursued among others by Balmino et al. (1973), Rapp (1982), Rummel et al. (1988) and Pavlis and Rapp (1990), while Tsoulis (2001) expanded the theory to include the Pratt/Hayford hypothesis. The investigation of the second possibility, using the global crustal information of the model CRUST 2.0 is the topic of the present paper.

3 Model 1: Moho-depths with a constant density contrast

The theoretical basis for the further development of equation (5) is the theory of isostasy: between crust and mantle there exists a condition of a floating equilibrium. Density anomalies are distributed in such a way, that this equilibrium is maintained. Below a certain depth, the so-called compensation depth, pressures are everywhere hydrostatic (Lambeck 1988).

In order to analyze the density and stratification information of the CRUST 2.0 model, an appropriate definition and/or adaptation of the definitions related to the t/i potential is necessary. If, for example the information of all seven crustal layers is to be taken into account, it is obvious that the simplified »two-layer« Earth model, used among others by Rummel et al. (1988) and Tsoulis (2001) for a formal computation of t/i coefficients, can not be applied. On the other hand, if only the information for the depth of the Mohorovicic discontinuity is to be exploited (the seventh layer of CRUST 2.0), one can use the conventional simplified Earth model, presuming that the crust having the constant density $\rho_{cr} = 2670\,\mathrm{kg\,m^{-3}}$ lies above the mantle with the density $\rho_m = 3270\,\mathrm{kg\,m^{-3}}$. In this section this two-layer simplified Earth model is applied, following the conventional approach for the t/i potential definition (e. g. Rummel et al. 1988). Furthermore, the mechanism entering here for the isostatic compensated Earth is the standard Airy/Heiskanen hypothesis. What remains to be settled is the choice for the compensation depth D, which will be addressed in the sequel. Between the two layers there exists a constant density contrast $\Delta\rho = \rho_m - \rho_{cr} = 600\,\mathrm{kg\,m^{-3}}$ (see Fig. 2). The potential coefficients corresponding to the isostatic equilibrium presented in Fig. 2 can be expressed as the difference between the coefficients corresponding to the surface topography and those generated by the compensation part. Equation (5) can now be rewritten as (Tsoulis 2001)

$$C_{lm}^{\alpha I} = \frac{3}{\bar{\rho}\bar{R}(2l+1)}\,\frac{1}{4\pi}\iint_{\sigma}\left[A^{T}(Q) - A^{C}(Q)\right]Y_{lm}^{\alpha}(Q)\,d\sigma_Q\,. \tag{6}$$

Equation (6) gives the so-called t/i potential harmonic coefficients of the Earth's gravity field. For the topography part it holds according to Fig. 2

$$A^{T}(Q) = \begin{cases} \displaystyle\int_{r=\bar{R}}^{\bar{R}+h}\left(\frac{r_Q}{\bar{R}}\right)^{l+2}\rho_{cr}\,dr_Q & \text{LAND PART} \\ \displaystyle\int_{r=\bar{R}-h'}^{\bar{R}}\left(\frac{r_Q}{\bar{R}}\right)^{l+2}[\rho_w - \rho_{cr}]\,dr_Q & \text{OCEAN PART} \end{cases} \tag{7}$$

with h and h' being the continental and the ocean part of the topography respectively (for CRUST 2.0 both taken from ETOPO5) and $\rho_w = 1030\,\mathrm{kg\,m^{-3}}$ an approximate value for the density of ocean water, adopted from CRUST 2.0. For the compensation part one arrives at

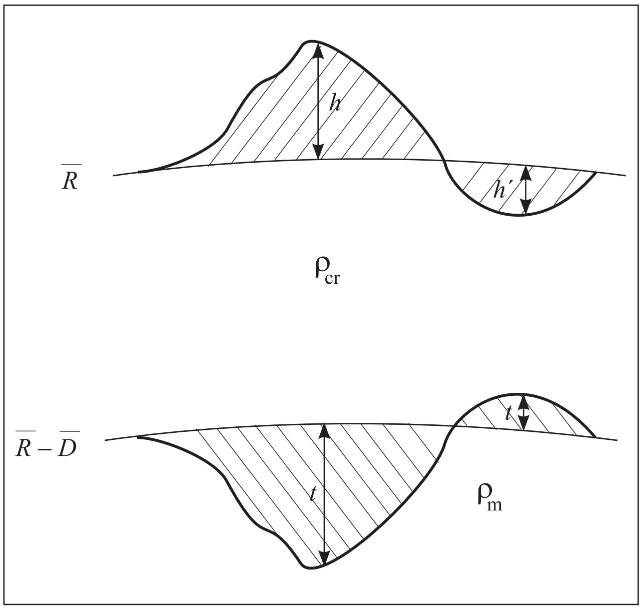

Fig. 2: Mohorovicic-information from model CRUST 2.0 combined with a constant density contrast hypothesis

$$A^{C}(Q) = \Delta\rho\int_{r=\bar{R}-\bar{D}-t}^{\bar{R}-\bar{D}}\left(\frac{r_Q}{\bar{R}}\right)^{l+2}dr_Q \tag{8}$$

where $\bar{D}$ stands for a mean value for the Mohorovicic discontinuity computed as a simple arithmetic mean over the seventh $2^\circ \times 2^\circ$ lower crust field of CRUST 2.0, described in the model as $t7$, $\Delta\rho$ the constant density contrast $\rho_m - \rho_{cr}$ and $t = |t7 - \bar{D}|$. The choice of $\bar{D}$ as the compensation depth is somewhat arbitrary and not necessarily physically justified. The numerical value of $\bar{D}$ as derived from the CRUST 2.0 data is 22.97 km. The root/antiroot thickness produced by this depth from the seventh CRUST 2.0 are compared in Tsoulis (2003), with the respective theoretically derived numerical values for the Airy/Heiskanen model using $D = 30$ km. The numerical comparison proved the value 22.97 km as the most suitable one in a global sense and it will be applied in the sequel.

Expanding the solutions to the one-dimensional integrations in equations (7) and (8)

$$\left[\left(\frac{r_Q}{\bar{R}}\right)^{l+3}\right]_{\bar{R}}^{\bar{R}+h},\ \left[\left(\frac{r_Q}{\bar{R}}\right)^{l+3}\right]_{\bar{R}-h'}^{\bar{R}}\ \text{and}\ \left[\left(\frac{r_Q}{\bar{R}}\right)^{l+3}\right]_{\bar{R}-\bar{D}-t}^{\bar{R}-\bar{D}} \tag{9}$$

into a binomial series up to third order in $h/\bar{R}$, $h'/\bar{R}$ and $t/\bar{R}$ respectively, yields in a few elementary steps after appropriate reordering

$$C_{lm}^{\alpha I} = C_{lm}^{\alpha\,T/LAND} + C_{lm}^{\alpha\,T/OCEAN} - C_{lm}^{\alpha\,C} \tag{10}$$

with

$$C_{lm}^{\alpha^{\mathrm{T/LAND}}} = \frac{3}{2l+1}\frac{\rho_{cr}}{\bar{\rho}}\left\{h_{lm} + \frac{l+2}{2}h2_{lm} + \frac{(l+2)(l+1)}{6}h3_{lm}\right\}, \tag{11}$$

$$C_{lm}^{\alpha^{\mathrm{T/OCEAN}}} = \frac{3}{2l+1}\frac{(\rho_w - \rho_{cr})}{\bar{\rho}}\left\{h_{lm} - \frac{l+2}{2}h2_{lm} + \frac{(l+2)(l+1)}{6}h3_{lm}\right\} \tag{12}$$

and

$$C_{lm}^{\alpha^{\mathrm{C}}} = \frac{3}{2l+1}\frac{\Delta\rho}{\bar{\rho}}\left\{\left(\frac{\bar{R}-\bar{D}}{\bar{R}}\right)^{l+2} t_{lm} - \frac{l+2}{2}\left(\frac{\bar{R}-\bar{D}}{\bar{R}}\right)^{l+1} t2_{lm} + \frac{(l+2)(l+1)}{6}\left(\frac{\bar{R}-\bar{D}}{\bar{R}}\right)^{l} t3_{lm}\right\}. \tag{13}$$

In equations (11), (12) and (13) a compact notation for certain surface harmonic expressions has been introduced, namely

$$h_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{h(Q)}{\bar{R}}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\,,$$

$$h2_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{h^2(Q)}{\bar{R}^2}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\,,$$

$$h3_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{h^3(Q)}{\bar{R}^3}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q \tag{14}$$

and

$$t_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{t(Q)}{\bar{R}}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\,,$$

$$t2_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{t^2(Q)}{\bar{R}^2}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\,,$$

$$t3_{lm} = \frac{1}{4\pi}\iint_\sigma \frac{t^3(Q)}{\bar{R}^3}\,\mathrm{Y}_{lm}^{\alpha}(Q)\,\mathrm{d}\sigma_Q\,. \tag{15}$$

For the sake of brevity, expressions (14) are involved both to the contribution of the land (equation (11)) and the ocean topography part (equation (12)). This technical detail should raise no misunderstandings: in both cases the Legendre spectrum of the same function is computed, namely that of global topography h. It is obvious that in the evaluation of the coefficients resulting from the land part of the topography (equation (11)), the spectra of h, h^2 and h^3 (equations (14)) are filtered out using an appropriate mask retaining only their continental part and vice versa for equation (12).

A spectral assessment of the t/i spectrum given by equation (10) is performed in Tsoulis (2003). Therein the t/i spectrum of Model 1 is compared with the EGM96 spectrum, truncated up to degree 90, the spectrum of the uncompensated topography and finally the spectrum obtained by the application of the standard Airy/Heiskanen scheme. It is shown that the derived t/i field possesses a rather undercompensated spectrum, mainly due to the lateral differences between the seventh CRUST 2.0 layer and the theoretically derived root-antiroot thickness according to the Airy/Heiskanen model.

Here an assessment in the space domain is performed, by evaluating global geoid undulations for given bandwidths of the computed model. The numerical investigations have shown namely, that the full set of t/i coefficients of Model 1 up to 90 degrees produces geoid undulations with respect to GRS80 reference ellipsoid that cannot be directly associated to known global geoid values. The undercompensated character of Model 1 returns to its full spectrum negative geoid undulations with a global mean value of ca. –2000 m. The restricted use of certain bandwidths of Model 1 gives, on the contrary, fields of global geoid undulations that can be associated to our common perception of a global geoid. Fig. 3 presents the geoid undulations computed from Model 1 for the selected coefficients $10 \le l \le 90$ and $0 \le m \le$ l. Fig. 4 shows furthermore the geoidal heights resulting from the EGM96 model for the same bandwidth. As Fig. 3 demonstrates, the field produced by the filtered Model 1 coefficients for this specific range, having minimum, maximum and mean values of –96.61 m, 93.29 m and –0.18 m respectively, contains a rather scaled contribution of the short wavelength part of the observed field. The respective quantity evaluated from the EGM96 model for the same coefficient range is much more damped and reveals the medium to high frequencies of the observed gravity field (Fig. 4). However, up to a certain level some kind of correlation between the two fields (Figs. 3 and 4) can be confirmed, especially in terms of the transition from positive to negative values and the overall structure of the obtained geoidal heights. Taking into account that for only the first several degrees the EGM96 field already reveals the contribution of the long wavelengths (e. g. up to degree 4, geoidal heights contributions of the order of magnitude between –70 m and 50 m), it is very interesting to gain new information regarding the medium and the higher frequencies of the gravity field.

4 Model 2: Analysis of all seven crustal layers

The exploitation of the total information included in the CRUST 2.0 database (stratification and density crustal data) is presented in Tsoulis (2003). After a proper redefinition of the Earth reference model as well as of the compensation mechanism describing the balance of the defined masses, a new set of t/i coefficients is derived (ibid., equations (1)–(4)). For the model producing the reference potential, a seven layer model Earth has been

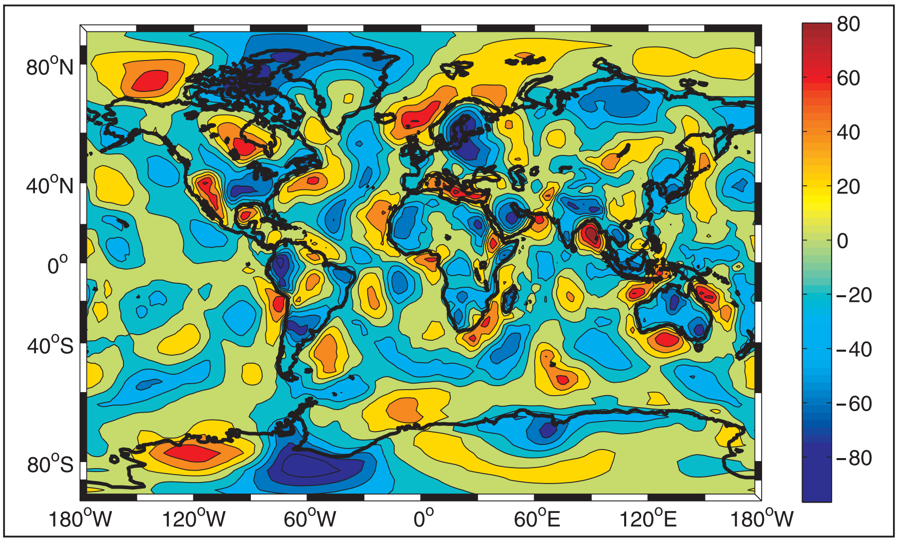

Fig. 3: Filtered geoid undulations with Model 1 ($10 \leq l \leq 90$), unit: m

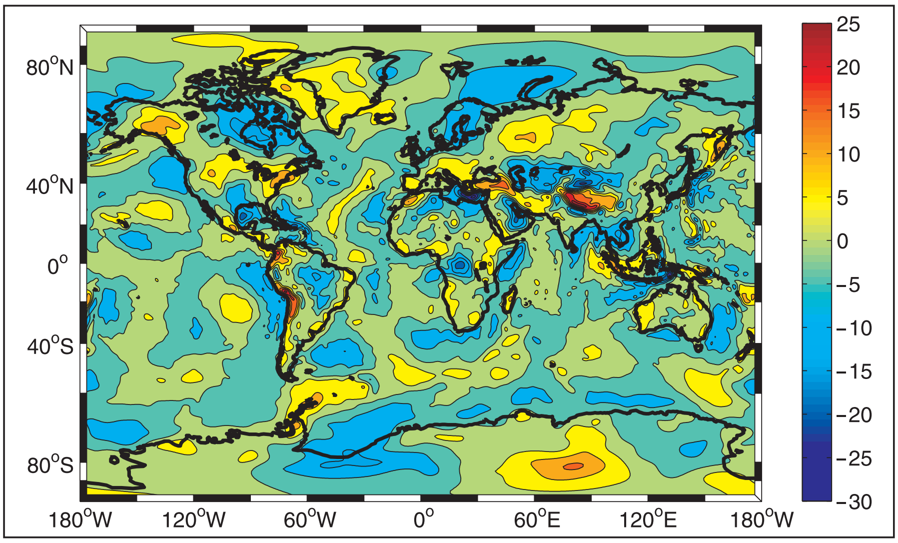

Fig. 4: Filtered geoid undulations with model EGM96 ($10 \leq l \leq 90$), unit: m

applied, based on the CRUST 2.0 data, as presented in Fig. 5. Furthermore, a compensation mechanism distributed radially along the seven crustal layers has been used to describe the isostatic equilibrium of the masses.

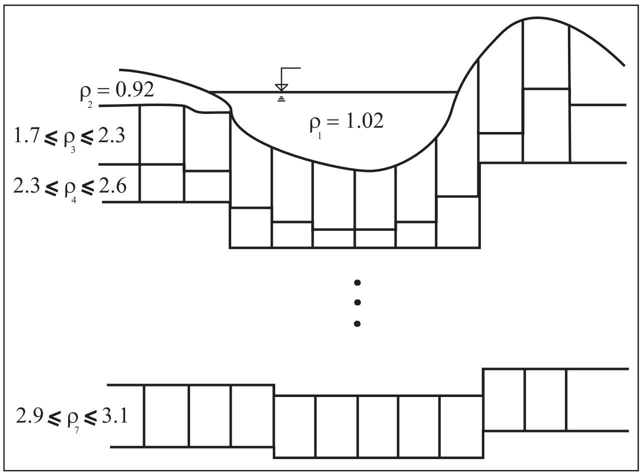

Fig. 5: The seven-layer simplified Earth model based on the density and stratification information of CRUST 2.0, density values in $g\,cm^{-3}$

Each of the seven CRUST 2.0 layers has a separate contribution to the compensation part, given for Model 1 simply from equation (13). The obtained t/i model, named in the sequel Model 2, shows a spectrum of rather reduced power with respect to that of the uncompensated topography. The compensation obtained by Model 2 is more obvious than the one attained from Model 1. However, taking the whole t/i spectrum into account, gives for Model 2 much more scaled geoidal heights (consistently negative values, global mean ca. −3200 m) than the ones computed from Model 1. Several coefficient bandwidths have been tested for Model 2 as well. Fig. 6 displays geoidal heights from Model 2 for the coefficient range $10 \leq l \leq 90$ and $0 \leq m \leq l$, applied already for Model 1. The obtained field is definitely a more scaled version of Fig. 3. Although in the spectral domain Model 2 appears to compensate the topographic signal more apparent than Model 1, the counterpart of Model 2 in the space domain reveals a much more under-compensated character. Although the overall structure of the field depicted in Fig. 6 resembles that of Model 1, the respective geoidal signals (minimum = −273.72 m, maxi-

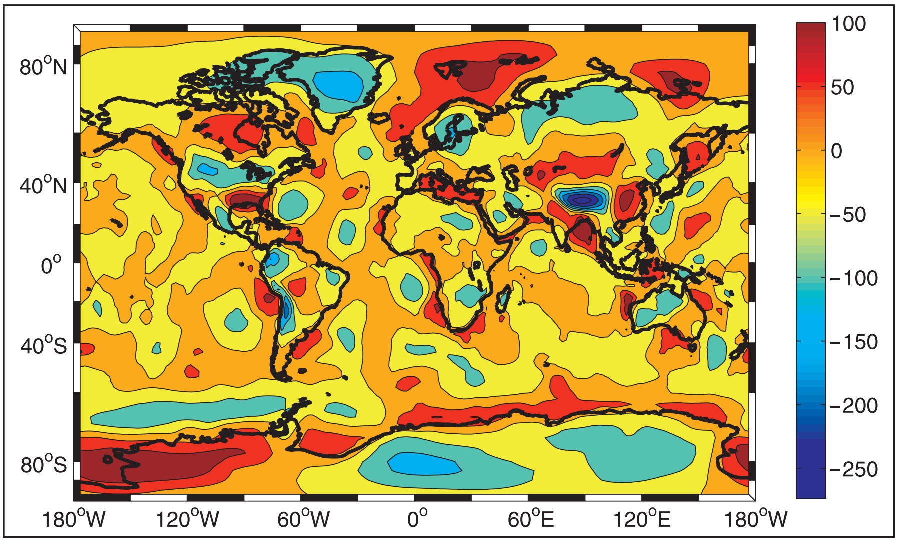

Fig. 6: Filtered geoid undulations with Model 2 ($10 \le l \le 90$), unit: m

mum = 145.95 m, mean = 0.73 m) indicate either the existence of long wavelength information in the CRUST 2.0 database, or the unsuitability of the applied radially distributed compensation mechanism.

5 Discussion

The availability of global digital databases for the crustal structure and the topography with an ever growing resolution creates some new assignments for the current geodetic research. Having the uppermost goal of a synthetic Reference Earth Model, one should first retrieve the gravity related information from existing datasets, such as the CRUST 2.0 global crustal database. This information combined with the new satellite only gravity models should cover the whole spectrum of the Earth's gravity field in a satisfactory manner. However, the task of interpreting the t/i models that result from the analysis of similar databases, or equally the tracking of bandwidths that can be geophysically interpretated, is a rather ambiguous problem. An efficient interpretation of the respective models should lie in the incorporation of further information regarding the Earth's interior and its distinctive discontinuities, such as the core-mantle boundary.

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