

A Methodology for the Identification of Periodicities in Two-dimensional Time Series

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Summary

A new technique to identify periodicities in two-dimensional movements has been investigated. As a typical example, polar motion is approximated by complex-valued series. The detected frequencies do not have to show integer ratios among themselves as in Fourier series; they can assume arbitrary real values. The fitting of the parameters of frequency series to the polar motion data results from a least-squares adjustment in which not only the amplitudes, but the frequencies as well are considered as unknowns. The approximate values for the frequencies in this severely nonlinear adjustment problem are obtained by a heuristic procedure. In doing so there is no need for prior information regarding individual possibly occurring frequencies. The methodology is illustrated by the Earth Orientation Parameter series C01 (EOP-C01) of the International Earth rotation and Reference system Service (IERS). The determined annual and Chandler frequencies lie inside the intervals determined by other authors using other techniques. However, since the frequencies determined by the new technique are not restricted to some set of Fourier-like frequencies, an interpretation of them as existing in the reality is less artificial. Therefore, the presented technique represents an expansion of the up to now available inventory of methods and can be successfully applied for the determination of frequencies contained in two-dimensional data. The advantages of the proposed procedure are that it can be easily generalized for the analysis of any time series of arbitrary dimensions and that even the data gaps within the analyzed time series would generate no theoretical or numerical difficulties.

Zusammenfassung

Es wurde eine neue Technik zur Identifizierung von Periodizitäten in zwei-dimensionalen Bewegungen entwickelt. Ein typisches Beispiel für derartige Bewegungen ist die Polbewegung, die durch eine komplexwertige Zeitreihe dargestellt werden kann. In diesem Fall müssen die Frequenzen keine ganzzahligen Verhältnisse zueinander aufweisen wie im Falle der Fourier-Reihen, sondern können beliebige Werte annehmen. Die Bestimmung der Parameter der komplexwertigen Frequenzreihe erfolgt mittels Ausgleichung, wobei nicht nur die Amplituden, sondern auch die Frequenzen als Unbekannte betrachtet werden. Die für diese nichtlineare Ausgleichung notwendigen Näherungswerte für die Frequenzen werden anhand eines heuristischen Verfahrens bereitgestellt. Aufgrund dieser Vorgehensweise besteht keine Notwendigkeit, Vorinformationen über eventuell vorhandene Frequenzen zu besitzen. Die Methodik des entwickelten Verfahrens wird anhand der Erdorientierungs-Parameter Serie C01 (EOP-C01) des International Earth rotation and Reference system Service (IERS) illustriert.

Dabei zeigt sich, dass die mittels des heuristischen Verfahrens ermittelten jährlichen Frequenzen und Chandler-Frequenzen innerhalb der Intervalle liegen, die von anderen Forschern unter Verwendung anderer Methoden bestimmt wurden. Allerdings ist eine Interpretation der heuristisch ermittelten Frequenzen in Bezug auf ihr Vorhandensein in der Realität mehr gerechtfertigt als bei der Verwendung einer eingeschränkten Anzahl von Fourier-Frequenzen. Deshalb kann die präsentierte Technik als Erweiterung aller derzeitigen Methoden betrachtet werden, mit der erfolgreich die Bestimmung von Frequenzen aus zweidimensionalen Daten möglich ist. Der Vorteil der vorgestellten Technik besteht darin, dass eine Generalisierung für die Analyse von Zeitreihen beliebiger Dimensionen möglich ist sowie auch das Vorhandensein von Datenlücken keine theoretischen wie numerischen Probleme mit sich bringt.

1 Introduction

The precise monitoring of polar motion and the understanding of the characteristics and time evolution of its periodic oscillations are important for defining the terrestrial reference frame and for geophysical studies (Höpfner 2003a). The study of earth's rotation is related to dynamic interactions between the solid earth, atmosphere, oceans and other geophysical fluids as well.

The complex motion of the deformable earth's rotational axis with respect to the crust of the earth exhibits secular, periodic and irregular variations due to several geophysical and meteorological causes. Some of them are still unexplained (Poma et al. 1997). One of the big challenges is the physical explanation of the big change of Chandler Wobble in the 1930ties (cf. Plag et al. 2005). A major recent progress in excitation studies of free wobble has been attained using the model proposed by Jeffreys (1940). Probably the best summary of the state of the art is given by Gross (2007).

However, there are still differing opinions regarding the Chandler frequency. Several authors found two or more close Chandler frequencies (e.g., Chandler 1901a, Gaposchkin 1972, Pan 2007) in the results of spectral analysis. Some authors believe that there is only a single free frequency changing with time (e.g., Carter 1981, Vondrák 1988, Pejović 1990, Höpfner 2003b) and many authors (probably the majority) argue that there is no evidence for temporal variation in the Chandler frequency (e.g., Wilson and Vicente 1981, Okubo 1982, Vicente and Wilson 1997). Already Chandler (1901a) has shown that the multiple-peaks model can be transformed into

a model with a single but variable frequency. Guo et al. (2005) show that two frequencies can be represented by a single frequency with a modulated amplitude and that it is not possible to distinguish both models based solely on mathematical arguments. Jochmann (2003) states that it cannot be resolved by analysis of time series whether the Chandler period is variable. Furthermore, he shows that it is impossible to differentiate phase from frequency modulation. An additional difficulty is that the Chandler wobble is probably not strictly harmonic, or even periodic. Hence, Jeffreys (1940) warns that the “analysis of an apparent free vibration as if it was strictly periodic is hazardous; its maintenance in spite of dissipation involves forces out of phase with the displacement, and these are not necessarily even periodic”.

All these difficulties do not justify ignoring the results from the analysis of observed data as long as there is no plausible physical explanation at hand. A possible approach is to determine the frequencies contained in the measured time series applying the multiple-peaks model. In next step the results can be transformed into a model with a single, but variable Chandler frequency and the significance of a variability can be investigated. For example Jochmann (2003) presented such an investigation, which resulted in the conclusion that the frequency of the Chandler wobble is in fact variable. However, due to the order of magnitude of these variations Chandler wobble can be treated as invariable for the length of time series presently available.

This article does not pretend to decide which of the models (multiple-peaks, single invariant or variable Chandler frequency) is physically better interpretable. It presents a new method for reliable determination of frequencies in two-dimensional time series within the multiple-peaks model. The method is illustrated by the example of the detection of secular polar motion and the most prominent periodicities from the Earth Orientation Parameter series C01 (EOP-C01) of the International Earth rotation and Reference system Service (IERS).

Usually some sort of Fourier analysis is applied for the evaluation of polar motion (and other two-dimensional) time series, where all frequencies employed to describe changes of the pole position are integer multiples of only one basic frequency. This basic frequency is most frequently chosen depending on the time interval covered by the time series, which has no physical justification. A better modeling of polar motion series tested for the long-periodic part can be found in Gross and Vondrák (1999). In this reference, the amplitude spectrum of polar motion series was not obtained by Fourier analysis, but was instead achieved by simultaneously fitting mean, trend and periodic terms to the observations. The fitting was repeated several times and the period of periodic term was systematically varied at intervals of 0.01 years between a period of 6 years and a period equal to the

length of series. The only shortcoming of such an approach is the enormous computational effort needed to perform the search at very small steps. This seems to be the main reason for the cited authors to apply this procedure only to the long-periodic part.

In this paper, a heuristic procedure is used for random search for frequencies within a chosen interval. In contrast to the systematic search applied in Gross and Vondrák (1999) it is possible to find arbitrary frequencies without prior knowledge about their existence. Furthermore, one needs significantly fewer parameters for the modeling of changes (e.g., of the pole position) described by the considered two-dimensional time series than in a standard Fourier-like approach.

The present paper focuses on the methodological aspect of the problem. The method is illustrated using the available IERS EOP-series C01. The same methodology can be applied in order to find periodicities in any other two-dimensional movement and can be straightforwardly generalized to arbitrary dimensions.

2 Mathematical Model for a Two-dimensional Movement

The polar motion of celestial intermediate pole (CIP) can be considered as an example of a two-dimensional movement. This motion is usually approximated by a complex-valued series (Munk and MacDonald 1960, Jochmann 1986, Jochmann and Felsmann 2001):

$$P(t) = x(t) + iy(t) = \sum_{k=-kmax}^{kmax} C_k \exp(i\Omega_k t). \quad (1)$$

Here, x and y denote the coordinates of the pole, t is the time, C_k is the amplitude, Ω_k an arbitrary (within the approach proposed in this study) angular frequency and $kmax$ the number of frequencies used. Since $C_k = A_k + iB_k$, this expression can be separated in two expressions for both pole coordinates. The resulting formulas have the following form:

$$x(t) = \sum_{k=0}^{kmax} A_{(k+)} \cos(\Omega_k t) - B_{(k+)} \sin(\Omega_k t) + A_{(k-)} \cos(\Omega_k t) + B_{(k-)} \sin(\Omega_k t), \quad (2)$$

$$y(t) = \sum_{k=0}^{kmax} A_{(k+)} \sin(\Omega_k t) + B_{(k+)} \cos(\Omega_k t) - A_{(k-)} \sin(\Omega_k t) + B_{(k-)} \cos(\Omega_k t). \quad (3)$$

$A_{(k+)}$ and $B_{(k+)}$ are the amplitudes of the prograde components of the polar motion, $A_{(k-)}$ and $B_{(k-)}$ the amplitudes of the retrograde components of the polar motion. The choice of $kmax$, apart from the obvious restrictions due to finite number of data points, should depend on the

desired quality of approximation of the time series, which is related with the accuracy of data. The frequencies Ω_k do not have to show integer ratios among themselves as in the case of Fourier series. They can obtain arbitrary real values, this is the essential difference between the considered expression (1) and the analogous expressions for Fourier series.

Each periodic part of the complex-valued series is equivalent to an elliptic component of the motion. This means that the pole goes along a deformed "ellipse" which is composed of many ellipses with various sizes and orientations of the semi-axes. The semi-axes can be determined from the following expressions:

$$a_k = \sqrt{A_{(k+)}^2 + B_{(k+)}^2} + \sqrt{A_{(k-)}^2 + B_{(k-)}^2}, \quad (4)$$

$$b_k = \sqrt{A_{(k+)}^2 + B_{(k+)}^2} - \sqrt{A_{(k-)}^2 + B_{(k-)}^2}. \quad (5)$$

The numerical eccentricity ε is a dimensionless measure for the deviation of the ellipse from the circle (Höpfner 2003a):

$$\varepsilon_k = \frac{\sqrt{a_k^2 - b_k^2}}{a_k}. \quad (6)$$

If ε is near to zero, the ellipse does not differ significantly from a circle; if ε is between zero and one, then it can be regarded as a real ellipse.

The two-dimensional movement and polar motion as an example for such a movement were chosen for this study in order to illustrate the proposed solution technique. The same procedure is applicable to any periodic phenomenon of arbitrary dimension, for instance to a periodic movement in three dimensions. It is only necessary to take a traditional Fourier-like mathematical model of the considered phenomenon and to replace the prescribed frequencies with arbitrary ones which become additional unknowns. The procedure described in the following sections can be directly adapted to the new problem. From the following description, it follows that no assumption has to be made on the distribution of data, it might be arbitrary. Examples of successful application of the method to one-dimensional time series with data gaps can be found in Mautz (2001, 2002).

3 Definition of the Problem

The problem which will be solved is the following: Given are the measured quantities $x + e_x$ and $y + e_y$, where e_x and e_y represent noise, the goal is to determine unknown amplitudes $A_{(k+)}$, $B_{(k+)}$, $A_{(k-)}$, $B_{(k-)}$ and unknown frequen-

cies Ω_k . By minimizing the sum of squared residuals a nonlinear least-squares adjustment for the determination of the unknowns is obtained. All observations can have either the same or different accuracies, which can be accounted for by using appropriate weighting factors. Solving the least-squares problem by local optimization techniques requires accurate approximate values for the frequencies, since the objective function has a lot of local minima. These approximate values are not known. However, bounds for the frequencies are available in case of equally spaced data (Priestley 1981):

- lowest possible frequency: $F_{min} = 1/p$,
- highest possible frequency: $F_{max} = 1/(2\Delta t)$.

The lowest possible frequency can be determined from the time span p of the considered time series based on the reasoning that the data interval should contain at least one full period in order to determine the frequency reliably. The highest possible frequency (Nyquist frequency) follows from the sampling rate Δt . Consequently, an interval can be specified which must contain all frequencies which can be determined using the considered data. By a heuristic procedure (which combines a random search for acceptable approximate values with a numerical solution of nonlinear least-squares adjustment) it is possible to find these frequencies which appropriately describe the changes of the considered functions, i.e., of pole coordinates.

In case of unevenly spaced data or data gaps the sampling rate is variable and the cited bounds cannot hold strictly; however, they can be used as guidelines.

4 Description of the Heuristic Procedure

In Fig. 1 the heuristic procedure is presented, see also Kaschenz (2003). This procedure can be divided into eight steps. First it is necessary to choose an arbitrary frequency Ω_k from the interval $[F_{min}:F_{max}]$, for example by using a random number generator. Secondly, with the chosen frequency it is possible to determine the amplitudes by linear least-squares adjustment as well as the corresponding residuals v_x and v_y (step 3). These residuals are the differences between the reduced (x_{red} , y_{red}) and the calculated (x_{calc} , y_{calc}) pole coordinates:

$$v_x = x_{red} - x_{calc},$$

$$v_y = y_{red} - y_{calc}.$$

The calculated coordinates are determined from equations (2) and (3) with $kmax=1$ using the chosen frequency and the corresponding amplitudes. At the beginning of the procedure the reduced pole coordinates are equal to the observed pole coordinates. In the fourth step all parameters are stored as preliminary solution. Now,

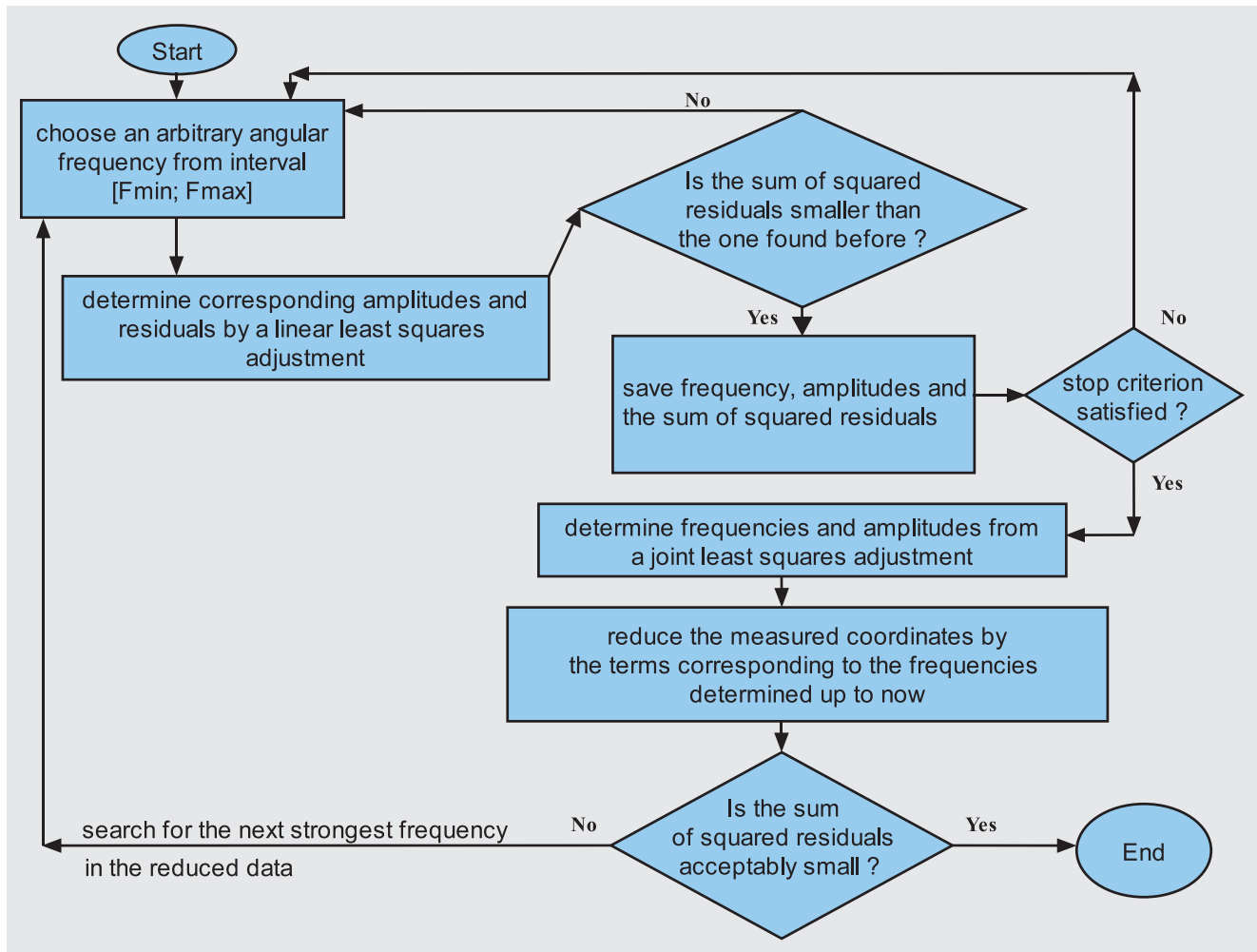


Fig. 1: Outline of the heuristic procedure

steps 1 till 3 are repeated several times and the obtained parameters are saved only in the case that the new sum of squared residuals is smaller than the last one memorized. The choice of the number of repetitions is not an easy question. At present, the criterion consisting of a mixture of maximum allowed number of repetitions (order of magnitude 10^6), number of “successes” (cases in which the new parameters are better than the old ones) and the magnitude of improvement in case of success seems to be the best compromise. Consequently, the saved new parameters describe the pole coordinates better than the parameters which have been stored before. After the repeated improvement of the unknown parameters, there is a good approximate value for a new significant frequency. Therefore, the determination of all up to then evaluated frequencies and the corresponding amplitudes within a common nonlinear least-squares adjustment is feasible. In the seventh step the observed pole coordinates are reduced by terms with periodicities determined up to then using equations (2) and (3) with k_{max} equal to the number of already evaluated frequencies. Within the reduced pole coordinates the search for the next strongest frequency is accomplished by going back to the first step.

The determination of frequencies stops if for example the (weighted) sum of squared residuals is acceptable. If the accuracy of the observed pole coordinates is known, an acceptable sum of squared residuals as a stop criterion can be determined from this information, since each value of this sum corresponds to some value of the mean approximation error resulting from least-squares adjustment. An alternative could be to postulate a significance level for amplitudes based on the noise level. Then, the procedure is either stopped or a prescribed number of frequencies is determined, of which only the significant ones are considered. Other criteria are imaginable as well, depending on the objective of investigation, which could be to approximate the data up to the known noise level, e.g., to identify a desired number of the strongest oscillatory components or something else.

It is important to notice that the search for frequencies achieved by that technique should not be regarded as really sequential, since after adding a new frequency, a strict nonlinear least-squares adjustment is performed by local optimization using all already determined frequencies as approximate values. In order to achieve (statistical) reliability that no adjustment within the procedure landed in a local minimum, the whole procedure is repeated several times.

5 Application to the EOP-C01 Series of the IERS

The capacity of the described technique is demonstrated using the polar motion series EOP-C01 of the IERS (<http://www.iers.org/products/38/11086/orig/eopc01>, 1900–now). We used the time span of 102 years, which is plotted in Fig. 2. The coordinates x and y of the Celestial Intermediate Pole are given relative to the IERS Reference Pole. The x -axis points towards the Greenwich meridian and the y -axis towards 90° West longitude.

The data values for x and y are a compilation of various sources and accompanied with estimated errors, having an average of 18 milli-arcseconds (mas). The more recent data are more accurate but it is not evident to assess the accuracy of the individual estimates. Using these error estimates for weighting would practically mean ignoring the major part of data. Since the main objective of this study consists in presenting the method, the data in this example are treated as equally weighted, which was also done in most studies analyzing the C01 series.

In addition to the periodic part an offset (a, c) and a drift parameter (b, d) for each pole coordinate are considered in order to take the polar wander into account (Schuh et al. 2001):

$$x(t) = a + b \cdot t + \sum_{k=0}^{k_{\max}} A_{(k+)} \cos(\Omega_k t) + \dots, \quad (7)$$

$$y(t) = c + d \cdot t + \sum_{k=0}^{k_{\max}} A_{(k+)} \sin(\Omega_k t) + \dots \quad (8)$$

This additional linear part produces no special problems and is determined within each common nonlinear least-squares adjustment where the frequencies are considered as unknowns, too. This is an advantage, since an independent determination of linear and periodic parts deteriorates the quality of result.

In the considered example 30 periodicities were determined, that is $k_{\max}=30$; some of these appear to be insignificant according to a noise-level criterion (average estimated error of 18 mas). For the chosen time series, the lowest possible frequency is $F_{\min}=0.0098/\text{year}$ and the highest possible frequency is $F_{\max}=10.0/\text{year}$.

The results for the linear part are:

$a = -3.236 \text{ as}$ and $b = 16 \text{ mas/year}$,

$c = -6.771 \text{ as}$ and $d = 35 \text{ mas/year}$,

or converted to the trend rate and trend direction:

trend rate = $\sqrt{b^2 + d^2} = 3.91 \text{ mas/year}$,

trend direction = $\arctan(d/b) = 65.13^\circ$ to the West longitude.

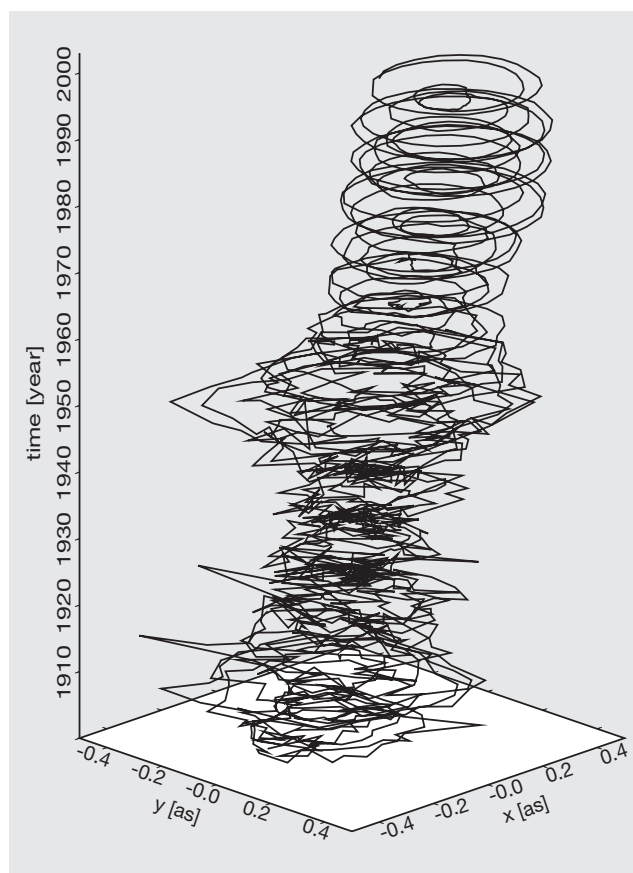


Fig. 2: EOP-C01 series of the IERS (1900–2002). The x -axis points towards the Greenwich meridian, "as" refers to arcseconds (roughly 30 m).

In Tab. 1 the trend rate and trend direction determined by several authors are confronted with the results obtained for these parameters in the present paper. The general agreement is rather good keeping in mind that the majority of other researchers do not determine those parameters simultaneously with the determination of frequencies of periodic parts (cf. e.g. Wilson and Vicente 1980, Vondrák et al. 1995, MacCarthy and Luzum 1996).

Tab. 1: Secular polar motion according to different sources resulting from various time series (http://hpiers.obspm.fr/eop-pc/models/PM/PM_secular_tab.html)

Source	Time span	Trend rate (mas/year)	Trend direction ($^\circ$ W long.)
Wilson and Vicente (1980)	1900–1977	3.40	66.0
Vondrak et al. (1995)	1900–1990	3.31	78.1
MacCarthy and Luzum (1996)	1899–1994	3.33	75.0
Gross (1998)	1899–1992	3.51	79.2
Vondrak (1999)	1899–1998	3.29	75.7
Schuh et al. (2001)	1899–1992	3.31	76.1
this article	1900–2002	3.91	65.1

Tab. 2: Periods of possible annual and Chandler wobbles

Annual wobble [years]	Chandler wobble [years]
1.0004	1.1931
1.0595	1.1742
1.0216	1.1553
0.9692	1.2318
1.0127	1.1923
0.9553	1.2123
	1.2449
	1.1634
	1.1050
	1.1226
	1.1866
	1.1361
	1.2226
	1.2966

The polar wander is largely caused by post-glacial rebound, although other mechanisms such as mantle convection and secular changes in ice sheet mass accompanied by a secular change in sea level have an influence (Gross and Vondrák 1999).

Frequencies expected on the basis of theoretical considerations (Moritz and Mueller 1987) are annual wobble with a period of around one year and the so-called Chandler wobble with a period of around 1.2 years. By using the described technique 6 slightly different possible “annual wobbles” and 14 slightly different possible “Chandler wobbles” were obtained. These are listed in Tab. 2 ordered by decreasing amplitudes (presented in Fig. 3a and 4a). It should be noticed that the criteria for separability of slightly different frequencies (Kovács 1980, Horne and Baliunas 1986) have little relevance when using the proposed method, since they are derived from the assumption that the method can determine only Fourier-like frequencies.

Fig. 3 shows that the strongest frequency corresponding to the annual wobble has a period of ~ 1.0004 years

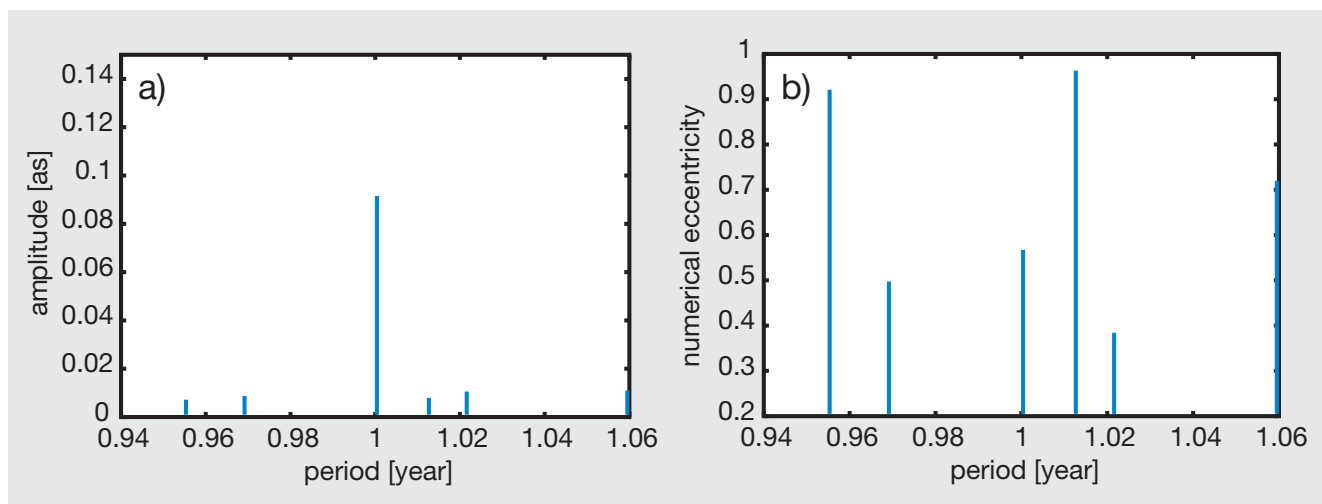


Fig. 3: Determined possible annual wobbles: a) amplitude spectrum; b) numerical eccentricity spectrum

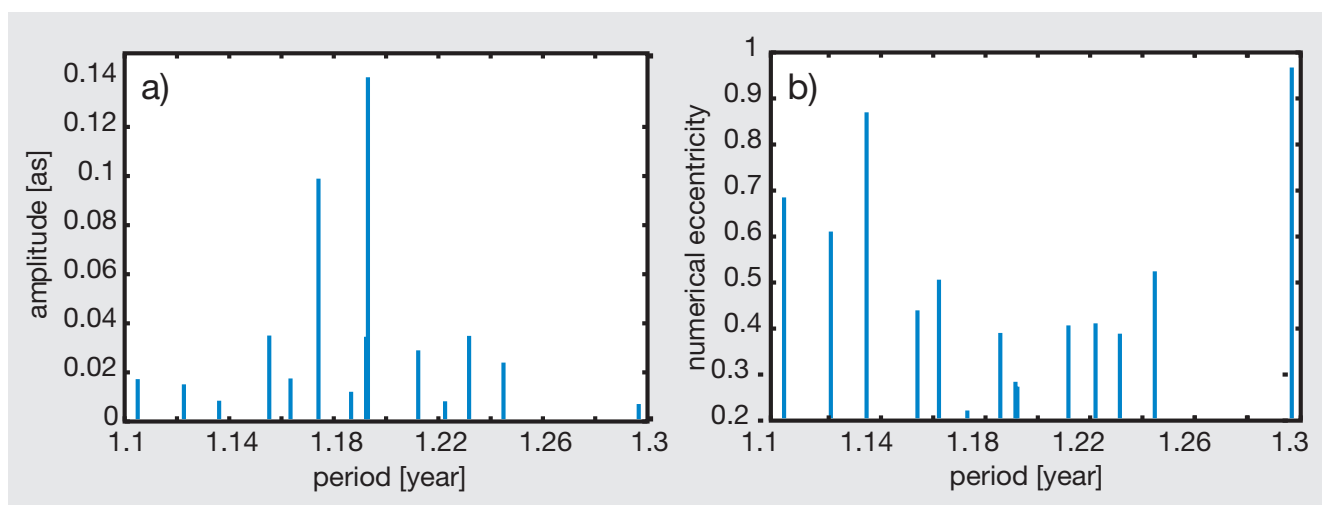


Fig. 4: Determined possible Chandler wobbles: a) amplitude spectrum; b) numerical eccentricity spectrum

with an amplitude of 90 mas. The respective numerical eccentricity is 0.56, which means that the annual wobble corresponds to an elliptic motion. The amplitudes of all other periods of around 1.0 year are smaller than 18 mas so that the determined strongest frequency can be interpreted as the “real” annual wobble.

Fig. 4 shows that the two strongest frequencies corresponding to the Chandler wobble have a period of ~ 1.19 and ~ 1.17 years with an amplitude of 140 mas and 100 mas respectively. The respective numerical eccentricities are the two lowest with 0.27 and 0.22. Thus the corresponding motion might be considered as approximately circular. The 12 remaining “Chandler wobbles” can be regarded as insignificant.

As already revealed, two or more frequencies close to the Chandler and annual periods could be found, similar to other authors’ results. Some authors believe that there are only two single frequencies which change over time, and also some argue that there is no evidence for temporal variations in frequencies (Vondrák 1988). In fact, both interpretations are imaginable.

Tab. 3 shows a comparison between the strongest obtained annual wobble using the described technique and the ones obtained by several other researchers. The difference between the result of Schuh et al. (2001) and the one presented here is only 0.05 days; both lie inside the interval determined by Höpfner (2002). In the same way a comparison between the two strongest obtained Chandler wobbles and results obtained by several other authors is demonstrated in Tab. 4. The periods of the two strongest frequencies are located within the intervals which were obtained by other researchers.

Taking into account that the presented technique needs no prior information about the existence or magnitude of frequencies and that any arbitrary real value for the frequencies is admissible, the obtained results for the Chandler and annual wobble represent the information contained in the data better than the results obtained using prior information or some hypothesis. Of course, the problem, whether the data reflect only the physical reality or contain additionally systematic errors, remains.

Altogether 30 frequencies were determined where 6 frequencies could be interpreted as “annual wobbles” and 14 frequencies as “Chandler wobbles”. As already discussed, only one of the determined annual wobbles and two of the Chandler wobbles have amplitudes above the noise level. Among the remaining detected periods, only three have periods larger than ten years and are listed in Tab. 5. Generally assumed cause of the first period is the climate cycle. The second period is the so-called Markowitz wobble for which the causes are unknown. The third period might be the double sunspot cycle. It seems that there is a significant 21-year periodicity in hydrometeorological processes correlated with this cycle (cf. Alexander 2005). The credibility of the frequencies

Tab. 3: Periods of annual wobble according to different sources

Source	Annual wobble [years]
Schuh et al. (2001)	1.00055
Höpfner (2002)	0.96–1.02
this article	1.0004

Tab. 4: Periods of Chandler wobble according to different sources

Source	Chandler wobble [years]
Chandler (1901b)	1.16–1.20
Vondrak (1988)	1.11–1.20
Schuh et al. (2001)	1.13–1.20
Höpfner (2002)	1.16–1.20
this article	1.1742 and 1.1931

Tab. 5: Periods larger than 10 years

Period [years]	Amplitude [as]	Assumed causes
70.684	0.0202	climate cycle
29.502	0.0109	Markowitz wobble
21.777	0.0072	double sunspot cycle

determined for the Markowitz wobble and for the double sunspot cycle is questionable, since the amplitudes lie somewhat beneath the average noise level.

6 Conclusions

The considered technique is a useful tool if one is interested in the identification of periodicities in time series. The heuristic technique for solving this global optimization problem has the following main advantages:

- for the frequencies any arbitrary real value is possible,
- the application to time series of any dimension is possible,
- the presence of data gaps within the analyzed time series generates no theoretical or numerical difficulties.

A disadvantage of the heuristic procedure is that the determination of the approximate values for the frequencies is a time-consuming procedure. This can be smoothed out by using an efficient strategy for the choice of arbitrary frequencies in the heuristic procedure.

The new method detects the frequencies contained in the data without any prior information or hypothesis. Hence, they are nearly an exact solution of the associated global nonlinear least-squares problem. The method

was illustrated using the two-dimensional EOP-C01 time series. The Chandler period as well as the annual period could be determined. Based on the large time span of 102 years, it is possible to detect periods longer than ten years and the components of the secular polar motion.

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